FRC Formation and Translation Simulations using the NIMROD Code

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Abstract

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The Plasma Science and Innovation Center (PSI-Center) is benchmarking and refining the NIMROD code for simulations of field-reversed configurations (FRCs). Recent modifications to the radial boundary conditions capture most of the effects of multiple discrete coils found in many FRC experiments. With this enhancement combined with the ability to include Hall physics, we have begun testing the ability of the code to predict FRC formation and translation, as well as toroidal field generation due to non-symmetric formation.

In several experiments including PHD, TCS-U and FRX-L, an FRC is formed in a theta-pinch section of the experiment and then it is translated axially. In the PHD experiment a conical translation section compresses the FRC during the translation phase. In TCS-U, an option exists to form a hot FRC in the theta-pinch section and then translate and expand it into the sustainment chamber. In FRX-L an FRC is formed and then translated into a compression chamber. We will report on recent progress made in simulating these experiments.

While the translation simulations have been initialized with an FRC equilibrium, we have also begun to study theta-pinch formation with the NIMROD code. One of the goals of this study is to understand the generation of toroidal magnetic field that has been observed in translated FRCs. It is thought that this toroidal magnetic field is due to the Hall effect and the non-symmetric formation that is often applied to FRCs prior to translation. In addition, we will be investigating the possibility that this process can also lead to the generation of poloidal flow.
Outline

◆ NIMROD Overview (Equations and boundary conditions)
◆ Equilibrium solver for translation calculations
◆ Translation calculations in a cylindrical geometry (thruster)
◆ PHD Stage 1 translation calculations
◆ Formation Calculations
NIMROD equations

Continuity
\[ \frac{\partial n}{\partial t} + \nabla \cdot (nu) = 0 \]

Momentum
\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \mathbf{J} \times \mathbf{B} - \nabla P - \nabla \cdot \Pi \]

Temperature
\[ \frac{n_s}{\gamma - 1} \left( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) T_s = -P_s \nabla \cdot \mathbf{u}_s - \Pi_s \cdot \nabla \cdot \mathbf{u}_s - \nabla \cdot q_s + Q_s \]

Faraday’s Law
\[ \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \]

Generalized Ohm’s Law
\[ \mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} (\mathbf{J} \times \mathbf{B} - \nabla P_e) \]
Boundary conditions

- A set of external coils can be simulated by specifying an axially varying $E_\theta$ on the radial boundary.

- Velocity: $v_\perp = \frac{E \times B}{B^2}$

- Allow an arbitrary number of coils.
  - Use a tanh function to interpolate between coils
  - Voltage time history $\sim \cos\left\{(t-t_f)/t_{1/4} \cdot (2/\pi)\right\}$

- Use either Dirichlet boundary condition or natural mass conserving boundary condition on density.
A new flexible equilibrium solver was developed, using “Mimetic Operators” (George Marklin), and extended for FRC equilibria.

**Grad-Shafranov equation**

\[-\Delta^* \psi = r^2 P'(\psi)\]

Assuming \(\psi = 1\) at the null, pressure specified as:

- **inside the FRC:** \(P(\psi) = P_s + \frac{(1 - P_s)}{(1 + \gamma)} (\psi + \gamma \psi^2)\)
- **on open field lines:** \(P(\psi) = P_s e^{(\delta \psi + \alpha \psi^2)}\)

Make \(P'\) continuous at separatrix: \(P_s = \frac{1}{1 + \delta(1 + \gamma)}\)
Equilibrium solver

In open field line region away from the FRC, ramp $P$ and $P'$ to zero so Grad-Shafranov equation becomes vacuum field equation.

Pressure (solid color) and flux lines for an *equilibrium* in a shaped flux conserver. Pressure is ramped off to the right of the FRC.
Electrodeless Lorentz Force (ELF) Thruster

- Currently simulating the acceleration of an FRC plasma in a straight drift tube.
- A travelling magnetic wave is simulated with a set of 29 magnets with a 5 cm axial spacing.
- The magnets are fired sequentially with a variable timing to account for the FRC acceleration.

\[
t_n = \sqrt{\frac{2(z_n - z_0)}{a}} - c
\]

- Coil at \( z = z_n \) is fired at \( t = t_n \). An acceleration \( a \) is assumed.
ELF calculation setup

- Start with an FRC in equilibrium inside a cylinder with a uniform flux.
- Fire coils sequentially behind the FRC to accelerate it to the right.
- Sinusoidal voltage waveform is applied. \( V_{coll} = V_0 \cos \left( \frac{\pi (t - t_{fire})}{2 \frac{t_{1/4}}{t_{1/4}}} \right) \)
Sample calculation parameters

- $B_{zBias} = 0.02$ T,
  - $V_o = 2500, t_{1/4} = 4 \times 10^{-6}, (\Delta B=0.1 \text{ T})$

- $n_o = 7.4 \times 10^{19} \text{ m}^{-3}, T_e=T_i \sim 30 \text{ eV}$

- $\eta/\mu = 10 \text{ m}^2/\text{sec}$ (About classical for 25 eV)
- $\nu = 500 \text{ m}^2/\text{sec}$

- Finite element grid = 16 x 72, with 4th order polynomial fit
Sample ELF calculation field and density

Density (solid color) + flux contours
ELF calculation time history

FRC mass

Trapped Flux

< V_z >

Axial Momentum
High density ELF calculation

- Represent a more massive FRC with a much higher density, rather than assuming a higher atomic mass (increase density by factor of 16)
- Use the same plasma transport (viscosity and resistivity)
- Slow the rate of the *travelling wave* by increasing the time between firing coils.
High density ELF calculation field and density

Density (solid color) + flux contours
High density ELF calculation time history

- FRC mass
- Trapped Flux
- $\langle V_z \rangle$
- Axial Momentum
Pulsed High Density (PHD) calculations

- We have begun simulations FRC translation in the PHD front-end
  - Not including compression into the burn chamber yet
- Start with a large $x_s$ FRC in the formation chamber
  - Representing plasma state shortly after lift-off

- These are the 1\textsuperscript{st} NIMROD FRC calculations with a shaped flux conserver with external coils.
PHD calculation parameters

◆ Computational region is 10 m x variable radial dimension (0.4 m max).

◆ Computation grid: 128 x 16 cells with 4<sup>th</sup> order polynomials

◆ Initial plasma conditions:
  – Temperature = 150 eV
  – Peak density = 2.6x10<sup>20</sup> (~ 6 mTorr fill)

◆ The large radius coils are <i>fired</i> sequentially with 30 kV and \( t_{1/4} = 13 \, \mu\text{sec} \) (0.5 T \( \Delta B \))

◆ Resistivity: \( \eta/\mu = 10 \, \text{m}^2/\text{sec} \) (Lunquist no. \( \sim 10^4 \))

◆ Viscosity: \( v = 500 \, \text{m}^2/\text{sec} \) (Reynold’s no. \( \sim 200 \))
PHD calculation density and field

Density (solid color) + flux contours
PHD calculation time history
FRC formation calculations

- Begin with uniform pressure vacuum field solution
  - Flux interpolation between coils, is same as for $E_\theta$.

- Use a high resistivity to annihilate flux near experimentally observed rates $\eta/\mu = 250 \text{ m}^2/\text{sec}$

- Viscosity: $\nu = 500 \text{ m}^2/\text{sec}$ (Reynold’s no. ~ 200)

- Use a $16 \times 48$ cell grid (4th order polynomial)

- Hall term is included in formation calculations
FRC formation calculation parameters

- $n_o = 2 \times 10^{20}$ (3 mTorr fill)
- $B_{bias} = 0.07$ T
- Apply voltage to center coil:
  - $V_o = 30$ kV, $t_{1/4} = 13$ $\mu$sec. ($\Delta B = 0.5$ T)
- Initial temperature ($T_o = 5$ eV)
Density evolution during formation

Density (solid color) + flux contours
Induced toroidal velocity during symmetric formation

$V_\theta$ (solid color), range is $-2 \times 10^4$ to $+4 \times 10^4$, + flux contours
Induced toroidal magnetic field during symmetric formation

$B_\theta$ (solid color), range is -0.05 T to +0.05T, + flux contours
Time evolution of key parameters during formation

- FRC flux and mass
- Temperature
- $B_{\text{Ext}}$
- $\Delta \Phi$
Non-symmetric formation calculations

- Segment the central coil into 3 sections
  - Fire each segment sequentially, to push the FRC axially
Density evolution during non-symmetric formation

Density (solid color) + flux contours

-t = 0.10 μsec
-t = 1.67 μsec
-t = 3.35 μsec
-t = 4.98 μsec
-t = 6.70 μsec
-t = 8.29 μsec
-t = 9.99 μsec
-t = 11.62 μsec
-t = 13.30 μsec
-t = 15.00 μsec
Induced toroidal velocity during non-symmetric formation

\( V_\theta \) (solid color), range is \(-2 \times 10^4\) to \(+4 \times 10^4\), + flux contours
Induced toroidal magnetic field during non-symmetric formation

$B_\theta$ (solid color), range is -0.05 T to +0.05 T, + flux contours
Time evolution of key parameters during non-symmetric formation
The NIMROD code is being employed to simulate a variety of FRC phenomena including:

- an FRC based thruster
- FRC translation and compression in the PHD experiment
- $\theta$-pinch formation

These studies are just beginning, and refinement of the boundary conditions is still required.

Formation studies including the Hall and $\nabla P_e$ terms will allow us to determine whether terms can explain observed toroidal magnetic fields.