MH4D Development

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Outline

• Summary of previous work
• New b.c. development
  – Non-parallel periodic boundaries
  – Operator matrix b.c.
• Atomic physics development
  – Spheromak simulations
• Other physics development
  – Variable resistivity, ohmic heating
• Benchmarking
  – Screw pinch instability
  – ZaP simulations
• Future work
Summary of previous work
Previous Work

- Insulating b.c.
  - Apply magnetic field b.c.
  - Allow non-zero $E_{\text{tang.}}$; $E_{\text{normal}} = 0$
  - Allow $v_{\text{normal}}$ (density floor required)
- Periodic b.c.
  - Multi-directional
- Initial ZaP simulations
New boundary condition development
Non-parallel periodic b.c.
Rotation required across boundary

Curl computation uses vector information from surrounding tetrahedra

+rotation  
- rotation

“Retained” side  
“Redundant” side
Non-parallel periodic b.c.
Operator matrices must be modified

Normally: \[ Cx = b \]

For rotated vectors \( x' \) and \( b' \):
\[
CR^T x' = R^T b' \quad \Rightarrow \quad RCR^T x' = b'
\]
\[ \Rightarrow C' = RCR^T \]

For boundary rotation and periodic rotation:
\[
CR_p^T R_b^T x' = R_p^T R_b^T b' \quad \Rightarrow \quad R_b R_p C R_b^T R_p^T x' = b'
\]
\[ \Rightarrow C' = R_b R_p C R_p^T R_b^T \]
**Operator matrix b.c.**

**Conducting boundary point →**

- Self-coupling is identity for tangential components.
- Neighbors couple only to normal component.

$$
\begin{align*}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & n \\
\end{align*}
\begin{align*}
\times & \times & \times \\
\times & \times & \times \\
\times & \times & \times \\
\end{align*}
= 
\begin{align*}
\times \\
\times \\
\times \\
\end{align*}
\begin{align*}
b \\
b \\
b \\
\end{align*}

**Insulating boundary point →**

- Self-coupling is identity for normal components.
- Neighbors couple only to tangential components.

$$
\begin{align*}
t & t & 0 \\
t & t & 0 \\
0 & 0 & 1 \\
\end{align*}
\begin{align*}
\times & \times & \times \\
\times & \times & \times \\
\times & \times & \times \\
\end{align*}
= 
\begin{align*}
\times \\
\times \\
\times \\
\end{align*}
\begin{align*}
b \\
b \\
b \\
\end{align*}
$$
Atomic physics development
Atomic Physics Development

• Phase I
  – Modify continuity equations
    • Assume constant neutral background density
    • Allow variable neutral background density

• Phase II
  – Modify momentum and energy equations
Temperature dependent ionization

\[ \frac{\partial \rho_i}{\partial t} = -\nabla \left( \rho_i \mathbf{v}_i \right) + \Gamma_{ion} \frac{\text{kg}}{\text{m}^3 \text{s}^{-1}} \]

\[ \Gamma_i = \langle \sigma_{ion} \mathbf{v}_e \rangle \rho_i n_n \]

Use \[ \langle \sigma_{ion} \mathbf{v}_e \rangle = \frac{2 \times 10^{-13}}{6.0 + \frac{T_{e, eV}}{13.6}} \left( \frac{T_{e, eV}}{13.6} \right)^{1/2} \exp \left( -\frac{13.6}{T_{e, eV}} \right) \text{ m}^3 \text{s}^{-1} \]

(ref. Goldston, Rutherford, Intro. to Plasma Physics, 1995, p. 151)

If \( \mathbf{v}_i = 0 \), analytically: \[ \rho_i = \rho_{i,0} \exp \left( \langle \sigma_{ion} \mathbf{v}_e \rangle n_n t_{total} \right) \]
Atomic Physics Development
Phase I

Temperature dependent recombination

\[ \frac{\partial \rho_i}{\partial t} = -\nabla (\rho_i v_e) - \Gamma_{rec} \]

\[ \Gamma_{rec} = \langle \sigma_{rec} v_e \rangle \rho_i n_i \]

\[ = \langle \sigma_{rec} v_e \rangle \rho_i^2 \frac{1}{1.67 \times 10^{-27}} \]

Use approximation: \[ \langle \sigma v_e \rangle = 0.7 \times 10^{-19} \left( \frac{13.6}{T_{e,v}} \right)^{1/2} \text{ m}^3 \text{s}^{-1} \]

(ref. Goldston, Rutherford, Intro. to Plasma Physics, 1995, p. 152)

If \( v_i = 0 \), analytically: \[ \rho_i = \frac{1}{t_{total} \langle \sigma_{rec} v_e \rangle \frac{1}{1.67 \times 10^{-27}} + \frac{1}{\rho_0}} \]
Track neutral fluid density

Assuming $v=0$,

\[
\begin{align*}
\frac{\partial \rho_n}{\partial t} &= -\Gamma_{\text{ion}} + \Gamma_{\text{rec}}; \\
\frac{\partial \rho_i}{\partial t} &= \Gamma_{\text{ion}} - \Gamma_{\text{rec}}
\end{align*}
\]

Compute $\Gamma_{\text{ion}}$ and $\Gamma_{\text{rec}}$ in each cell. Must not violate simple accounting!

- if $\Gamma_{\text{ion}}\,dt$ is greater than $\rho_{\text{ion}}$, $\rho_{\text{ion}} = \rho_{\text{ion}} + \rho_n$; $\rho_n = 0$
- if $\Gamma_{\text{rec}}\,dt$ is greater than $\rho_i - \rho_{i,\text{floor}}$, $\rho_n = \rho_n + \left( \rho_i - \rho_{i,\text{floor}} \right)$;

$\rho_i = \rho_{i,\text{floor}}$
Comments

- MH4D gives precisely the expected analytical solutions for simple problems.
- In coronal equilibrium, MH4D predicts the expected ~50% ionization at 1.5 eV.
- Predicts the expected ~100% ionization at 13.6 eV.
Single-fluid plasma model
- Cold neutral fluid with initial distribution
- Assume $v_n = 0$

$$\frac{\partial \rho_i}{\partial t} = -\nabla (\rho_i v_i) + \Gamma_{ion} - \Gamma_{rec}$$

$$\frac{\partial \rho_n}{\partial t} = -\Gamma_{ion} + \Gamma_{rec}$$

$$\rho_i \frac{\partial v_i}{\partial t} + \nabla (p_e + p_i) = j \times B - \left( \Gamma_{rec} + \Gamma_{cx} \right) m_i v_i$$

$$\frac{\partial p}{\partial t} + v \nabla p = -\gamma p \nabla v + (\gamma - 1) \eta j^2 - (\gamma - 1) \left( \Gamma_{ion} + \Gamma_{cx} \right) m_n \Phi_{ion}$$

$$- \left( \Gamma_{rec} + \Gamma_{cx} \right) m_i p$$
• Spheromak tilt problem
  – “Tuna can” flux conserver
  – L/R=2.5
  – Beta=2.5%
  – Flat pressure profile
  – Force–free i.c. $\rightarrow B$
    • Find $A$ that satisfies $\text{curl}(A)=B$
    • Allow $A_{\text{tangential}}$, on boundaries, but $d/dt(A_{\text{tang.}})=0$.
  – Find tilt growth rate with and without atomic physics
Atomic Physics Development
Phase II

Movie
Atomic Physics Development
Phase II

Total kinetic energy (J)

Plasma mass increases with time
99.97% ionized

K.E. growth with and without atomic physics

Tilt growth rate is 14.0% less with atomic physics.

\[ \gamma_\tau_A = 0.047 \text{ vs. } \gamma_\tau_A = 0.055 \]
Other physics development
Variable resistivity,
Ohmic heating

• Chodura (anomalous) resistivity:

\[ \eta_c = v_c \frac{m_e}{ne^2}, \quad \nu_c = C_c \omega_{pi} \left[ 1 - \exp \left( -\frac{v_e}{f\nu_s} \right) \right], \quad \nu_e = \frac{j}{ne}, \quad \nu_s = \sqrt{\frac{\gamma p}{\rho}} \]

\[ C_c \approx 1, \quad f \approx 3 \]

• Spitzer resistivity:

\[ \eta_{sp.} = \left( 5 \times 10^{-5} \right) \ln \Lambda \]

\[ \frac{T_{eV}^2}{3} \]

• Explicit timestep limit:

\[ 2 \frac{\eta}{\mu_0} \frac{\Delta t}{\Delta x^2} < 1 \]

• Ohmic heating:

\[ \frac{\partial p}{\partial t} + \nabla p = -\gamma p \nabla \cdot \mathbf{v} + (\gamma - 1) \eta \mathbf{j}^2 \]
Chodura resistivity

![Graph showing Chodura Resistivity with cap and floor, and increasing j.]
Benchmarking
Screw pinch benchmark

- Parabolic pressure profile
- $T=\text{const.}=100$ eV
- $B_z=\text{const.}$
- Density $\sim p/kT$ (but use density floor)
- Helical velocity perturbation
Screw pinch benchmark, cont.’d

Movie

1

5.000000e-10
Screw pinch benchmark

Growth rate comparison

\[ \gamma \tau_A \]

- MH4D, q=0.5
- MH4D, q=0.7
- MH4D, q=0.9
- linstab, q=0.5
- linstab, q=0.7
- linstab, q=0.9

\[ dx, \text{ meters} \]
Spheromak benchmark

MH4D tilt growth rates

Published data* indicates 0.17

Comparison to MACH2 via ZaP simulations

• Work in progress
• Resolution unlikely to approach MACH2
• Interplay of several factors is complicating matters:
  – Significant resistivity range is needed
  – High resolution is necessary
  – Low density floor would be ideal to capture low density current paths in plasma
Future work
Future work

• To finish MS:
  – Implement implicit and semi-implicit methods for high resolution sim’s?
  – Atomic physics in ZaP
  – Significant time available on Bassi

• PhD work:
  – To be discussed shortly by Dr. Shumlak