Verification of a Hall MHD Algorithm for NIMROD with the FRC Tilt Mode

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Abstract

The accurate and efficient computation of low frequency two-fluid phenomena in a confined plasma remains a challenge. As an initial project of the Plasma Science and Innovation Center (PSI-Center), a time-implicit two-fluid version of NIMROD\textsuperscript{1} is being verified and applied to the macroscopic stability of a FRC. FRC macro-stability is a problem of intrinsic interest, both from a fundamental and from a practical standpoint, and also presents a unique verification opportunity. Initial $n = 1$ linear results have reproduced earlier observed\textsuperscript{2,3} transition from the “fundamental” MHD internal tilt mode to modes with higher structure along $B$, with growth rates smaller but comparable to MHD growth rates. Previous long-thin analysis\textsuperscript{4} has been extended to capture the features of these modes and results are compared with NIMROD. Modifications to the analysis suggest effects beyond HMHD, which are important for the stability of these modes. Such effects will be incorporated into future NIMROD versions and verified by comparison with the analysis.

\begin{itemize}
  \item \textsuperscript{1} C.R. Sovinec et al. J. Comp. Phys. \textbf{195}, 355 (2004)
  \item \textsuperscript{2} R.D. Milroy, D.C. Barnes, R.C. Bishop, and R.B. Webster, Phys. Fluids \textbf{B1}, 1225 (1989)
  \item \textsuperscript{3} Elena V. Belova, Ronald C. Davidson, Hantao Ji, and Masaaki Yamada, Phys. Plasmas \textbf{10}, 2361 (2003)
  \item \textsuperscript{4} D. C. Barnes, Phys. Plasmas 10, 1636 (2003)
\end{itemize}
Background: The FRC Tilt Mode

- Most dangerous global mode for FRCs.
- $n=1$
- Is not easily verified by experiment.
- Actual growth times are longer than predictions of resistive MHD.
- Two-fluid, FLR and kinetic effects slow the mode’s growth.
Background: The FRC as a Tool for Code Development

- FRCs provide stringent verification of the reliability of FLR and kinetic effects in numerical simulations
- Highly kinetic device with large Larmor radii where the Hall term becomes potent
- High Beta
- Large density gradients
- Simply connected
Previous FRC tilt work

• Previous numerical work on Hall MHD
  – Stabilization of “fundamental” (rigid shift per $\psi$ surface)
  – “Higher” parallel structure modes with fraction of MHD growth rate
• Long-thin analysis shows
  – “Perfect” agreement with MHD and HMHD fundamental
  – No higher modes
• To resolve dichotomy, need
  – Better codes (improve NIMROD 2-fluid to allow big $\Delta t$)
  – Better theory (has to agree with codes)
FRC Results

- Use Barnes long-thin equilibrium
- MHD baseline
- 2-fluid with varying ion density
Barnes long-thin FRC Equilibrium (5:1)
NIMROD 1&2-fluid Reproduces Previous Numerical FRC Result

Non-Equilibrium Pressure

Weakly Kinetic Hall MHD, viscosity=100 m^2/s,
electric diffusivity= 100 m^2/s, density=1.4e21 m^-3, E/S*=.22, γ = .965 \( \gamma_{MHD} \)
NIMROD 1&2-fluid Reproduces Previous Numerical FRC Result

Z Component of Velocity

Weakly Kinetic Hall MHD, viscosity=100m^2/s, electric diffusivity=100m^m/s,
density=1.4e21 m^-3, E/S^*=.22, γ = .965 \gamma_{MHD}
Effect of Hall Physics on Growth

![Graph showing the effect of Hall Physics on growth rate. The graph plots growth rate versus $E/S^*$ with different runs labeled: Run A, Run B, Run C, and Run D. The graph also indicates the resistive MHD line.](image_url)
Hall MHD Model of Tilt Mode Growth

- Extension of MHD to include $\vec{J} \times \vec{B}$ and neglect $\nabla P$ gives the dispersion relation:

$$\omega^2 + \frac{a \gamma_{MHD}}{S^* / E} \omega + \gamma_{MHD}^2 = 0$$

- Solving for the growth rate gives:

$$\gamma^2 = \gamma_{MHD}^2 \left[ 1 - \left( \frac{a}{S^* / E} \right) \right]$$

Hall Theory for Large E/S*

![Graph showing Hall MHD, Extended Hall MHD, and Realistic Diffusion]
Poloidal Structure in Pressure Seen for Moderate E/S*

Non-Equilibrium Pressure

Moderately Kinetic Hall MHD, viscosity=2 m^2/s,
electric diffusivity= 0.2 m^2/s, density=1.25e20 m^-3, E/S*=.721, γ = .52 γ_{MHD}
Poloidal Structure in Velocity Seen for Moderate E/S*

Z Component of Velocity

Moderately Kinetic Hall MHD, viscosity=2 m^2/s, electric diffusivity= 0.2 m^2/s, density=1.25e20 m^-3, E/S* = .721, γ = .52 γ_{MHD}
High Poloidal Structure in Pressure
Seen for Highest E/S*

Non-Equilibrium Pressure
Highly Kinetic Hall MHD, viscosity=2 m^2/s,
electric diffusivity= 0.2 m^2/s, density=6.25e19 m^-3, E/S*=1.02, γ = .44 \gamma_{MHD}
High Poloidal Structure in Pressure Seen for Highest $E/S^*$

Non-Equilibrium Pressure
Highly Kinetic Hall MHD, viscosity=2 m^2/s, electric diffusivity= 0.2 m^2/s, density=6.25e19 m^-3, $E/S^*$=1.02, $\gamma = .44 \gamma_{MHD}$
High Poloidal Structure in Velocity Seen for Highest E/S*

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Comparison of Poloidal Velocity Profiles

- Examine the profile in the region adjacent to the “elbow” and in the high E/S* region
- Isotropic flow is seen in the Hall MHD region
- Some velocity shear is seen in the extended Hall MHD region just past the elbow
- Smaller vorticity structures are seen in the region of high E/S*
- Vectors are plotted in the plane of maximum velocity
\[ V(\theta=63.025, t=1e-05) \quad V_{\text{max}} = 1e+06 \]

\[ E/S^* = 0.684 \]
$\theta=234$, $t=1e-05$ $V_{max} = 6.88e+05$

$E/S^* = .791$
\[ V(\theta=143.98, t=1e-05) \quad V_{\text{max}} = 1.96e+06 \]

\[ E/S^* = 1.02 \]
Summary

- Higher poloidal structure seen by NIMROD
- Hall MHD mode structure is not dissimilar to the MHD mode
  - Probably a negative energy mode destabilized by coupling to positive energy radial displacement mode
- Region of high E/S* indicates that new physics is needed
- Dominant modes in the high E/S* region appear to be coupled, in contrast to the assumptions of the previous theory
Future work

- Introduce non-linear Hall MHD to NIMROD
- Change to fully implicit time stepping algorithm
- Analyze the character of the tilt mode in the regime of large potency of the Hall term
- Contrast any differences between modes in the Hall MHD and extended Hall MHD regions
- Develop a more complete theory to explain the growth in the region of strong E/S*
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