Boundary conditions for physical fluxes in plasmas

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- Interactions & Reactions Considered
- Single Plasma Fluid and Neutral Fluid Model
- Required Boundary Conditions
- Physical Model for Boundary Conditions

Plasma model that includes neutral gas effects on plasma evolution.

Kinetic description is most appropriate, but not feasible for the Center's goals. Instead we derive a fluid model that captures the essential physics. The derivation follows that by Braginskii and generalizes to include the effect of atomic reactions. Assumptions:

- Fluid model provides an adequate description for the plasma.
- Plasma is represented by a three-component plasma – electron, ion, neutral fluids.
- All excited states are lumped into the ion or neutral species.
- Additional assumptions are identified as they are made.

Only a subset of interactions & reactions are considered.

For typical ICC plasmas, the most important atomic reactions are electron impact ionization, radiative recombination, and charge exchange.

$$e^- + n \rightarrow i^+ + 2e^- - \phi_{ion}$$

$$e^- + i^+ \rightarrow n + h\nu$$

$$i^+ + n \rightarrow n + i^+$$

where ϕ_{ion} is the ionization potential and $h\nu$ is the photon energy. Non-reacting collisional interactions are handled in the usual manner.

Fluid model is derived from moments of the Boltzmann equation.

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \bullet \frac{\partial f_\alpha}{\partial \mathbf{x}} + \frac{q_\alpha}{m_\alpha}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \bullet \frac{\partial f_\alpha}{\partial \mathbf{v}} = \left.\frac{\partial f_\alpha}{\partial t}\right|_{collisions} = \sum_\beta C_{\alpha\beta}$$

Collisional term (RHS) accounts for all reactions. Any collisions of species β that affects species α. Including atomic reactions leads to

$$\int C_{\alpha\beta} d\mathbf{v} \neq 0.$$

Moment equations for each species

The first three moments of the Boltzmann equation gives equation for species continuity, momentum, and total energy. For example, the equations for the ion species are

$$\frac{\partial n_i}{\partial t} + \nabla \bullet \left(n_i \mathbf{v}_i \right) = \Gamma_{ion} - \Gamma_{rec}$$

$$\frac{\partial}{\partial t}\left(m_i n_i \mathbf{v}_i \right) + \nabla \bullet \left(m_i n_i \mathbf{v}_i \mathbf{v}_i + p_i \mathbb{I} \right) = q_i n_i \left(\mathbf{E} + \mathbf{v}_i \times \mathbf{B} \right) + \mathbf{R}_{ie} + \mathbf{R}_{in}$$
$$+ m_i \mathbf{v}_n \Gamma_{ion} - m_i \mathbf{v}_i \Gamma_{rec} + \left(m_n \mathbf{v}_n - m_i \mathbf{v}_i \right) \Gamma_{cx}$$

$$\frac{\partial \varepsilon_i}{\partial t} + \nabla \bullet \left(\left(\varepsilon_i + p_i \right) \mathbf{v}_i + \mathbf{h}_i \right) = \mathbf{v}_i \bullet \left(q_i n_i \mathbf{E} + \mathbf{R}_{ie} + \mathbf{R}_{in} \right) + Q_{ie} + Q_{in}$$
$$+ \frac{1}{2} m_n v_n^2 \Gamma_{ion} - \frac{1}{2} m_i v_i^2 \Gamma_{rec} + \left(\frac{1}{2} m_n v_n^2 - \frac{1}{2} m_i v_i^2 \right) \Gamma_{cx}$$

where Γ_{ion} is the rate of ionization and is dependent on species densities and temperatures.

Fluid model includes single ionized fluid and single neutral fluid.

The three fluid model can be reduced to a single plasma fluid and a neutral fluid. The model is comparable to MHD with a neutral fluid. The usual MHD assumptions are applied, $n_i = n_e$ and $m_e \to 0$.

$$\frac{\partial n_i}{\partial t} + \nabla \bullet \left(n_i \mathbf{v}_i \right) = \Gamma_{ion} - \Gamma_{rec}$$

$$\frac{\partial n_n}{\partial t} + \nabla \bullet \left(n_n \mathbf{v}_n \right) = \Gamma_{rec} - \Gamma_{ion}$$

The momentum equations for the plasma and neutral momenta are

$$\frac{\partial}{\partial t}\left(m_i n_i \mathbf{v}_i \right) + \nabla \bullet \left(m_i n_i \mathbf{v}_i \mathbf{v}_i - \mathbf{BB}/\mu_0 + \left(p_i + p_e + B^2/2\mu_0 \right)\mathbb{I} \right) =$$
$$\mathbf{R}_{in} + \mathbf{R}_{en} + m_n \mathbf{v}_n \Gamma_{ion} - m_i \mathbf{v}_i \Gamma_{rec} + \left(m_n \mathbf{v}_n - m_i \mathbf{v}_i \right)\Gamma_{cx}$$

$$\frac{\partial}{\partial t}\left(m_n n_n \mathbf{v}_n \right) + \nabla \bullet \left(m_n n_n \mathbf{v}_n \mathbf{v}_n + p_n \mathbb{I} \right) =$$
$$-\mathbf{R}_{in} - \mathbf{R}_{en} + m_i \mathbf{v}_i \Gamma_{rec} - m_n \mathbf{v}_n \Gamma_{ion} + \left(m_i \mathbf{v}_i - m_n \mathbf{v}_n \right)\Gamma_{cx}$$

Fluid model includes single ionized fluid and single neutral fluid.

The magnetic induction equation is similar to the one from MHD.

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \bullet \left(\mathbf{v}_i \mathbf{B} - \mathbf{B} \mathbf{v}_i \right) = \nabla \times \left(\frac{\mathbf{j} \times \mathbf{B} - \nabla p_e - \mathbf{R}_{ie} + \mathbf{R}_{en}}{e n_i} \right)$$

The energy equations for the total plasma energy and total neutral energy

$$\frac{\partial}{\partial t}\left(\varepsilon_i + \varepsilon_e \right) + \nabla \bullet \left(\left(\varepsilon_i + p_i \right) \mathbf{v}_i + \left(\varepsilon_e + p_e \right) \mathbf{v}_e + \mathbf{h}_i + \mathbf{h}_e \right) = \mathbf{j} \bullet \left(\mathbf{E} + \frac{\mathbf{R}_{ie}}{e n_i} \right) + \mathbf{v}_i \bullet \mathbf{R}_{in} + \mathbf{v}_e \bullet \mathbf{R}_{en}$$

$$+ Q_i + Q_e - \frac{1}{2} m_i v_i^2 \Gamma_{rec} + \left(\frac{1}{2} m_n v_n^2 - \phi_{ion} \right) \Gamma_{ion} + \left(\frac{1}{2} m_n v_n^2 - \frac{1}{2} m_i v_i^2 \right) \Gamma_{cx}$$

$$\frac{\partial \varepsilon_n}{\partial t} + \nabla \bullet \left(\left(\varepsilon_n + p_n \right) \mathbf{v}_n + \mathbf{h}_n \right) = -\mathbf{v}_n \bullet \left(\mathbf{R}_{in} + \mathbf{R}_{en} \right)$$

$$+ Q_n - \frac{1}{2} m_n v_n^2 \Gamma_{ion} + \left(\frac{1}{2} m_i v_i^2 - h\nu \right) \Gamma_{rec} + \left(\frac{1}{2} m_i v_i^2 - \frac{1}{2} m_n v_n^2 \right) \Gamma_{cx}$$

Boundary conditions for plasma fluid + neutral fluid model

Expressing the governing equations in compact form,

$$\frac{\partial Q}{\partial t} + \nabla \bullet F_{div} + \nabla \times F_{curl} = S$$

Integrating over the domain and applying vector calculus theorems,

$$\frac{\partial}{\partial t} \int_{\Omega} dV\, Q + \oint_{\partial\Omega} dS\, \mathbf{n} \bullet F_{div} + \oint_{\partial\Omega} dS\, \mathbf{n} \times F_{curl} = \int_{\Omega} dV\, S$$

Consistent boundary values must be specified for the appropriate flux terms in the surface integrals.

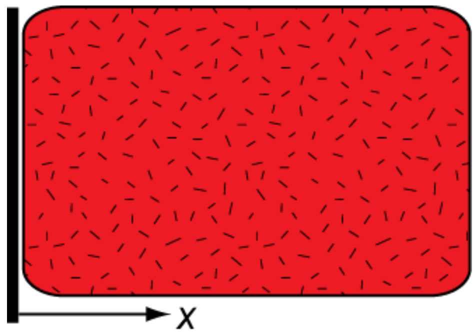
$$\mathbf{n} \bullet F_{div}$$

$$\mathbf{n} \times F_{curl}$$

Boundary conditions for particle fluxes

Consider the case of a solid wall that is impermeable to plasma but produces neutrals through outgassing and recombination.

Assume boundary to the left of plasma in x direction.



For the continuity equations, this gives

$$\mathbf{n}\bullet\left(n_i\mathbf{v}_i\right) \rightarrow n_i v_{ix}\Big|_{\partial\Omega} = -n_i v_{i_{Th}}$$

$$\mathbf{n}\bullet\left(n_n\mathbf{v}_n\right) \rightarrow n_n v_{nx}\Big|_{\partial\Omega} = \left(n_n v_{n_{Th}}\right)_{inj} - R n_i v_{ix}$$

Ions flow into the wall at the thermal speed and are recycled with a fraction of R.

Neutrals flow out of the wall due to injection/outgassing and due to recycling of the ions from recombination.

Boundary conditions for momentum fluxes

For the momentum equations, the ion momentum flux at the boundary is

$$\mathbf{n}\bullet\left(m_i n_i \mathbf{v}_i \mathbf{v}_i - \mathbf{BB} + \left(p + B^2/2\right)\mathbb{I}\right) \rightarrow m_i n_i v_{ix} \mathbf{v}_i\Big|_{\partial\Omega} - B_x \mathbf{B}\Big|_{\partial\Omega} + \left(p + B^2/2\right)\mathbf{x}\Big|_{\partial\Omega}$$

Consistent with the particle fluxes the following need to be specified:

$$n_i v_{ix} \mathbf{v}_i\Big|_{\partial\Omega} = -n_i v_{i_{Th}} \mathbf{v}_i; \qquad B_x \mathbf{B}\Big|_{\partial\Omega}; \qquad \left(p + B^2/2\right)\Big|_{\partial\Omega}$$

The neutral momentum flux at the boundary is

$$\mathbf{n}\bullet\left(m_n n_n \mathbf{v}_n \mathbf{v}_n + p_n \mathbb{I}\right) \rightarrow m_n n_n v_{nx} \mathbf{v}_n\Big|_{\partial\Omega} + p_n \mathbf{x}\Big|_{\partial\Omega}$$

The consistent fluxes that need to be specified are

$$n_n v_{nx} \mathbf{v}_n\Big|_{\partial\Omega} = \left(n_n v_{n_{Th}}^2\right)_{inj} \mathbf{x} - R n_i v_{ix} v_{n_{Th}} \mathbf{x}; \qquad p_n\Big|_{\partial\Omega}$$

Boundary conditions for energy fluxes

For the energy equations, the plasma energy flux at the boundary is

$$\mathbf{n}\bullet\left(\left(\varepsilon_i + p_i\right)\mathbf{v}_i + \left(\varepsilon_e + p_e\right)\mathbf{v}_e + \mathbf{h}_i + \mathbf{h}_e\right)$$

$$\rightarrow \left(\varepsilon_i + p_i\right)v_{ix}\Big|_{\partial\Omega} + h_{ix}\Big|_{\partial\Omega} + \left(\varepsilon_e + p_e\right)v_{ex}\Big|_{\partial\Omega} + h_{ex}\Big|_{\partial\Omega}$$

Consistent with the particle fluxes the following need to be specified:

$$\left(\varepsilon + p\right)v_{ix}\Big|_{\partial\Omega} = -\left(\varepsilon + p\right)v_{i_{Th}}; \qquad h_x\Big|_{\partial\Omega} = 0$$

The neutral energy flux at the boundary is

$$\mathbf{n}\bullet\left(\left(\varepsilon_n + p_n\right)\mathbf{v}_n + \mathbf{h}_n\right) \rightarrow \left(\varepsilon_n + p_n\right)v_{nx}\Big|_{\partial\Omega} + h_{nx}\Big|_{\partial\Omega}$$

The consistent fluxes that need to be specified are

$$\left(\varepsilon_n + p_n\right)v_{nx}\Big|_{\partial\Omega} = \left(\left(\varepsilon_n + p_n\right)v_{n_{Th}}\right)_{inj} - R\left(\varepsilon_n + p_n\right)v_{ix}; \qquad h_{nx}\Big|_{\partial\Omega} = 0$$

Boundary conditions for magnetic induction fluxes

For the magnetic induction equation, the associated fluxes at the boundary are

$$\mathbf{n}\bullet\left(\mathbf{v}_i\mathbf{B}-\mathbf{B}\mathbf{v}_i\right)\rightarrow v_{ix}\mathbf{B}\Big|_{\partial\Omega}-B_x\mathbf{v}_i\Big|_{\partial\Omega}$$

$$-\mathbf{n}\times\left(\frac{\mathbf{j}\times\mathbf{B}-\nabla p_e-\mathbf{R}_{ie}+\mathbf{R}_{en}}{en_i}\right)\approx\mathbf{n}\times\eta\mathbf{j}\rightarrow\eta\mathbf{j}_{\mathbf{t}}\Big|_{\partial\Omega}$$

Consistent with the particle fluxes the following need to be specified:

$$v_{ix}\mathbf{B}\Big|_{\partial\Omega}=-v_{i_{Th}}\mathbf{B};\qquad B_x\mathbf{v}_i\Big|_{\partial\Omega};\qquad \eta\mathbf{j}_{\mathbf{t}}\Big|_{\partial\Omega}$$