Continuum DKE closures for PSI-C applications

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Outline

1. Continuum DKE solutions for PSI-C applications.
   - Equation to be solved.
   - Efficient velocity space representation.
   - Linearized Coulomb operator for accurate collisional effects.
   - An example and conclusions.

Requirements for EC applications.

- Kinetic solution of DKE with closures tightly coupled to advance of fluid equations.
- Efficient velocity space representation.
- Accurate collisional effects using full linearized Coulomb collision operator.
- Drift approximation appropriate for electrons in many EC experiments and for ions in some cases.

Solve Chapman-Enskog-like (CEL) drift kinetic equation.

- Solve CEL-DKE:

$$\frac{\partial F}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla F + \frac{q}{m} E_{||} \frac{v_{||}}{v} \frac{\partial F}{\partial v} =$$

$$\langle C(F + f_M) \rangle - \frac{mv^2}{T} P_2(v_{||}/v) f_M \Big(\mathbf{bb} - \frac{\mathbf{I}}{3}\Big) : \nabla \mathbf{u} -$$

$$\frac{2 f_m}{3p} L_1^{(3/2)} [\nabla \cdot \mathbf{q} + \mathbf{\Pi} : \nabla \mathbf{V} - Q] +$$

$$\mathbf{v}_{||} \cdot \left[\frac{f_m}{p} (\nabla \cdot \mathbf{\Pi} - \mathbf{R}) + \frac{f_m}{T} L_1^{(5/2)} \nabla T \right] + \textit{drift drives}.$$

- Compute closures $q_{||}$, $\pi_{||}$, $\mathbf{R}_{||}$, and Q and couple to fluid equations at each time step.

Use 1D finite element basis for $v/v_{||}$.

- Rewrite CEL-DKE to emphasize $v_{||}/v$ dependence:

$$\partial_t F + v_{||}\mathbf{b}\cdot\nabla F + v_D\cdot\nabla F - C =$$
$$+f(\mathbf{r},v,t) + v_{||}g(\mathbf{r},v,t) + \left[3\left(\frac{v_{||}}{v}\right)^2 - 1\right]h(\mathbf{r},v,t)$$

- Expand F in finite element basis functions: $F(\mathbf{r},t,v_{||}/v,v) = f(\mathbf{r},v,t)\phi(x(v_{||}/v))$, where $\phi(x) = (\phi_0,\phi_1,...,\phi_p)^T$.
- Can use h and p refinement with grid packing near resonances in velocity space.

Allow for modal and nodal basis functions.

- Hierarchichal (modal) basis built from Legendre polynomials (Sprague and Geers, “Legendre spectral finite elements for structural dynamics analysis" , 2007):
  $\phi_0 = (P_0 - P_1)/2$
  $\phi_1 = (P_0 + P_1)/2$
  $\phi_i(x) = P_i - P_{i-2}$ for $i \epsilon \{2, 3, ..., p\}$.
- Nodal basis using Lagrange or Gauss-Labotto polynomials also available.

Solve coupled PDE system for FE coefficients.

- Coupling matrices, **M**'s, computed at startup and used in

$$\mathbf{M}_t \partial_t f + \mathbf{M}_L \nu_L f + (\mathbf{M}_s v \mathbf{b} + \mathbf{M}_{\mathbf{ExB}} \mathbf{V}_{ExB} + \mathbf{M}_{\nabla B} \mathbf{V}_{\nabla B} + \mathbf{M}_{\mathbf{J}} \mathbf{V}_J) \cdot \nabla f + (\mathbf{M}_{sB} v \mathbf{b} + \mathbf{M}_{ExBB} \mathbf{V}_{ExB} + \mathbf{M}_{JB} \mathbf{V}_J) \cdot \nabla \ln B f = \mathbf{F} + \mathbf{G} + \mathbf{H}$$

- Elements of stiffness matrices computed like $\mathbf{M}_L = -\int_{-1}^{1} dx J^{-1} \phi_i' (1 - (v_{||}/v)^2) \phi_j'$ where Jacobian is half length of 1D FE cells.

Moments computed using numerical quadrature.

- Can use Gauss-Laguerre quadrature for s integral:
  $M_{||}^{lk} = 2\pi \frac{l!}{(2l-1)!!} v_T^{l+2k+3} \sum_{js} \frac{1}{2} w_{js} s_{js}^{l+1-2n} e^{s_{js}^2} L_k^{l+1/2}(s_{js}^2) \sum_j P_{lj} f_j(\mathbf{r}, t, s_{js})$
- Gauss-Legendre quadrature on a truncated domain $s\epsilon[0, s_{max}]$:
  $M_{||}^{lk} = 2\pi \frac{l!}{(2l-1)!!} v_T^{l+2k+3} \sum_{js} w_{js} s_{js}^{l+2} L_k^{l+1/2}(s_{js}^2) \sum_j P_{lj} f_j(\mathbf{r}, t, s_{js})$.
- Equations solved at number of speed grid points, s_{js}, needed for accurate moments
  $q_{||} = 2\pi \int_{-1}^{1} d(v_{||}/v) \int_0^{\infty} dss^5()$ and
  $\pi_{||} = 2\pi \int_{-1}^{1} d(v_{||}/v) \int_0^{\infty} dss^4()$.

Coulomb collision operator.

- Use moment form (Ji and Held, PoP (2006)) for remainder of collision operator:
  $C^{aa} = \frac{1}{n_a v_{Ta}^{l+2k}} \sum_{lk} \frac{f_a^{(0)}}{\sigma_k^l} P_l(v_{||}/v) M_{||}^{lk}(\mathbf{r},t)\nu_{aa}^{lk*}.$
- Inserting moment definitions yields coupling between coefficients at different speeds

$$C_{ij}^{aa} = \frac{2}{\sqrt{\pi}} e^{-s_{is}^2} \sum_{js} w_{js} \begin{pmatrix} \frac{1}{2} s_{js}^{l+1-2n} e^{s_{js}^2} \\ s_{js}^{l+2} \end{pmatrix} L_k^{l+1/2}(s_{js}^2)$$

$$\sum_j \sum_{lk} \frac{l!}{(2l-1)!!} \frac{\nu_{aa}^{lk*}(s_{is})}{\sigma_k^l} P_{li} P_{lj} f_j(\mathbf{r}, t, s_{js})$$

Time discretization for collisions.

- Representative EC parameters: $n = 1 \times 10^{19}\, m^{-3}$ and $T = 100\, eV$.
- Collision times: $\tau_e = 2.37 \times 10^{-6}\, s$ and $\tau_i = 1.35 \times 10^{-4}\, s$.
- If $C \sim (1/\tau)\partial_s^2$, explicit treatment of the moment collision terms requires$\Delta t << \tau(\delta s)^2$.
- Explicit treatment of moment collision terms eases parallelization over speed points.
- Apply to sound wave damping problem.

Conclusions

- Kinetic solution of DKE with closures tightly coupled to advance of NIMROD's fluid equations.
- Efficient velocity space representation.
- Accurate collisional effects using full linearized Coulomb collision operator.
- For PSI-C applications, drift approximation appropriate for electrons and possibly for ions.
- Revisit SSPX transport calculations as initial test.
- Proceed to applications requiring full coupling of drift kinetic closures into fluid model.