Open boundary condition progress

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Motivation:

- To model infinite domains, open boundaries are often needed to limit the computational domain size without influencing the solution.
  - E.g. plasma accelerators, magnetic reconnection problems.
- A general open BC is needed that can handle
  - dissipative MHD
  - flows and waves oblique to boundaries.

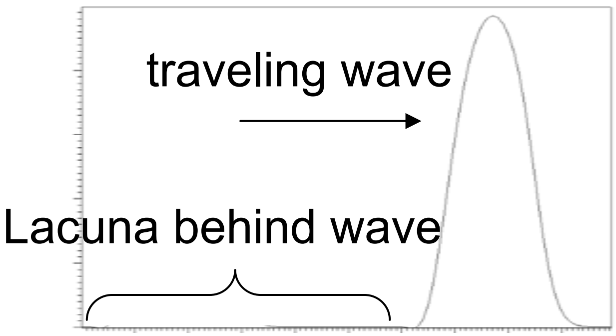
Outline:

- A new approach: Lacuna-based open BC (LOBC)*
- Navier-Stokes results
- Future plans

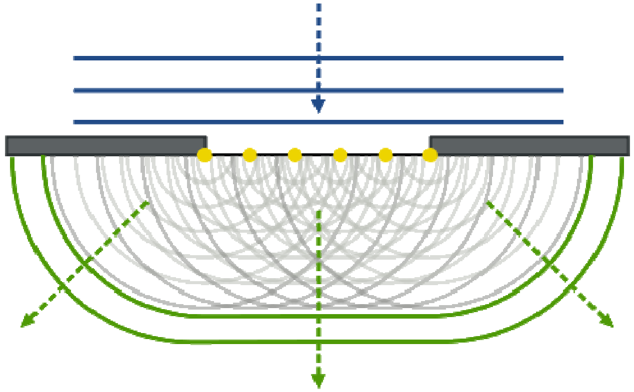
* The approach has been developed for single wave-speed hyperbolic systems by Ryaben'kii, Tsynkov et al. See V.S. Ryaben'kii, S.V. Tsynkov, V.I. Turchaninov, J. Comp. Phys. 174 (2001) 712.

Lacunae are still regions behind waves in hyperbolic systems

- Lacunae are easily observed in the 1D scalar wave equation.



- Huygens (1629-1695) used the concept of discrete propagation of individual wavelets to explain diffraction.



- As will be seen later, an important aspect of lacunae is that they are present only in odd-dimensional spaces.

A transition region is used to generate sources for an auxiliary problem (AP)

Interior problem: $\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial}{\partial x_i}\mathbf{F}^i(\mathbf{q}) = \mathbf{S}(\mathbf{q}).$

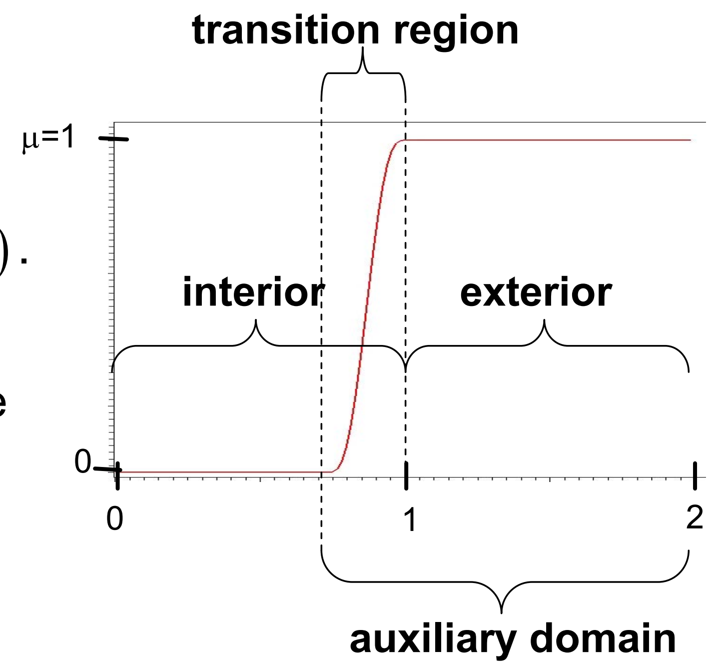
Auxiliary problem (AP): $\frac{\partial \mathbf{w}}{\partial t} + \frac{\partial}{\partial x_i}\mathbf{F}^i(\mathbf{w}) = \mathbf{S}(\mathbf{w}) + \mathbf{\Omega}(\mathbf{q}).$

To determine $\mathbf{\Omega}$, substitute $\mathbf{v} \equiv \mu \mathbf{q}$ for $\mathbf{w}$ and solve

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{\partial}{\partial x_i}\mathbf{F}^i(\mathbf{v}) = \mathbf{S}(\mathbf{v}) + \mathbf{\Omega}(\mathbf{q})$$

$$\frac{\partial \mathbf{v}}{\partial t} = \mu \frac{\partial \mathbf{q}}{\partial t} = -\mu \frac{\partial}{\partial x_i}\mathbf{F}^i(\mathbf{q}) + \mathbf{S}(\mathbf{q})$$

$$\Rightarrow \mathbf{\Omega} = \frac{\partial}{\partial x_i}\mathbf{F}^i(\mathbf{v}) - \mathbf{S}(\mathbf{v}) - \mu \frac{\partial}{\partial x_i}\mathbf{F}^i(\mathbf{q}) + \mathbf{S}(\mathbf{q})$$



AP is reintegrated, disregarding "lagging" source terms

- Transition sources drive AP solution to match interior solution at interior/exterior interface.

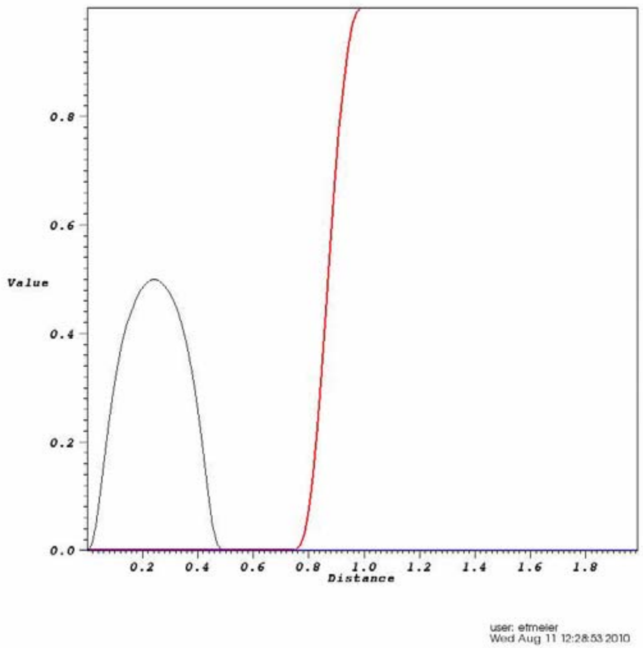
- Truncate "lagging" transition source terms.

  - Wave feature enters domain at $t = t_1$.
  - Define lacuna time, $t_L = L_{ext}/c$.
  - Effects will reach exterior boundary at $t = t_1 + t_L$
  - For $t_2 > t_1 + t_L$, reintegrate the AP,

$$\mathbf{w}\big|_{t_2} = \mathbf{w}\big|_{t_0} + \int_{t_1}^{t_2} \frac{\partial \mathbf{w}}{\partial t}\, dt$$

$$= \int_{t_1}^{t_2} \left(-\frac{\partial}{\partial x_i} \mathbf{F}^i(\mathbf{w}) + \mathbf{S}(\mathbf{w}) + \mathbf{\Omega} \right) dt$$

Scalar wave eqn with LOBC

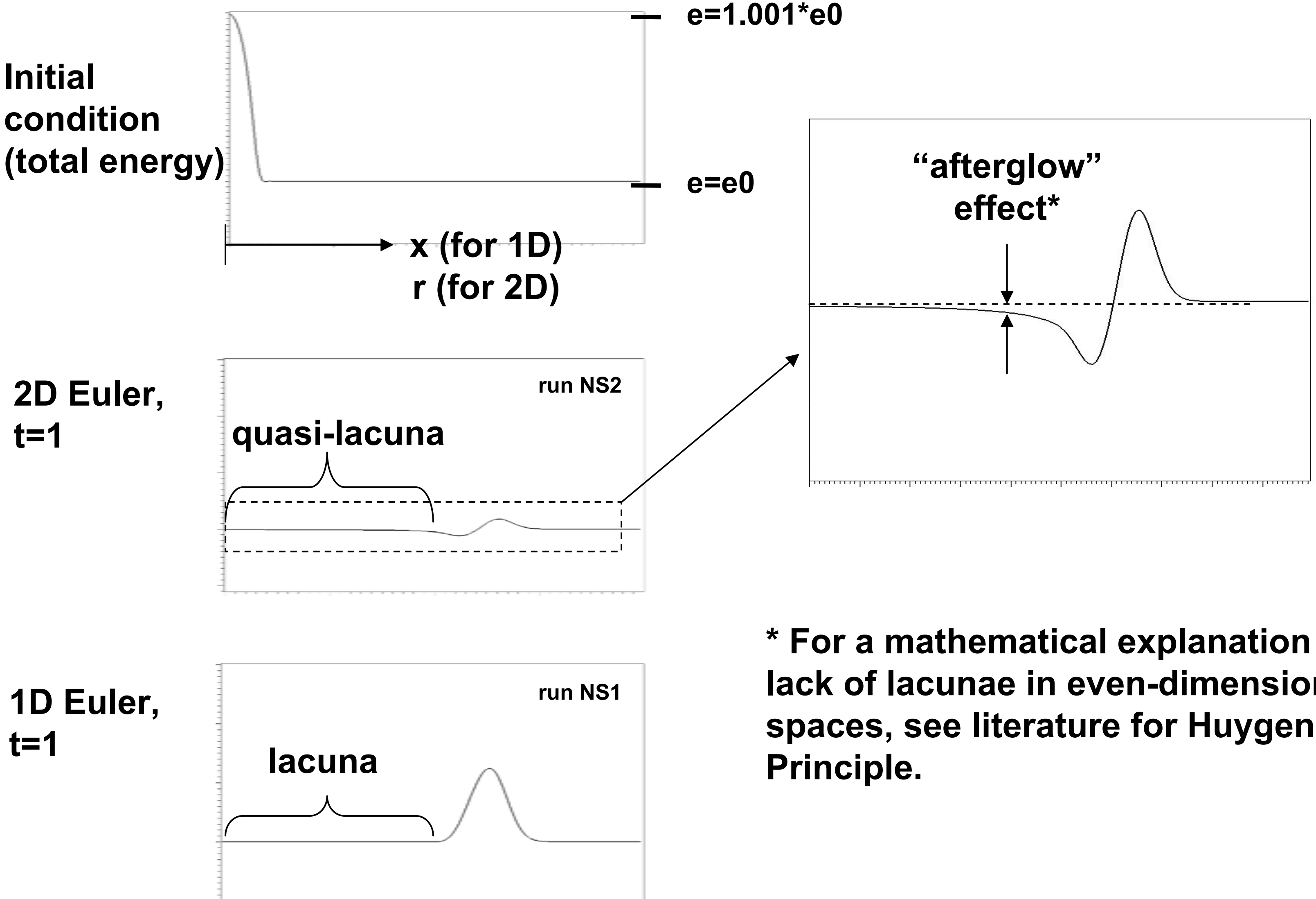


black = wave
blue = trans. source
red = trans. function, μ

LOBC provide perfect non-reflection only under certain circumstances.

- Necessary conditions for perfect non-reflection include
  i. The system described has odd spatial dimensionality.
  ii. The system is purely hyperbolic. With parabolic effects, lacunae are no longer present.
  iii. When multiple wave speeds are present, source terms related to the slowest wave must exit the transition region.
    - 1D Euler equations speeds: u, $u+c$, $u-c$. Slow flow speed at the boundary could be problematic.
- The method can provide an effective open boundary even when one or more of these conditions are not met.
- (i) and (ii) are addressed in the next slides. (iii) will be studied in future work.

In 2D, lacunae exist in an approximate sense (quasi-lacunae)



* For a mathematical explanation of lack of lacunae in even-dimensional spaces, see literature for Huygens’ Principle.

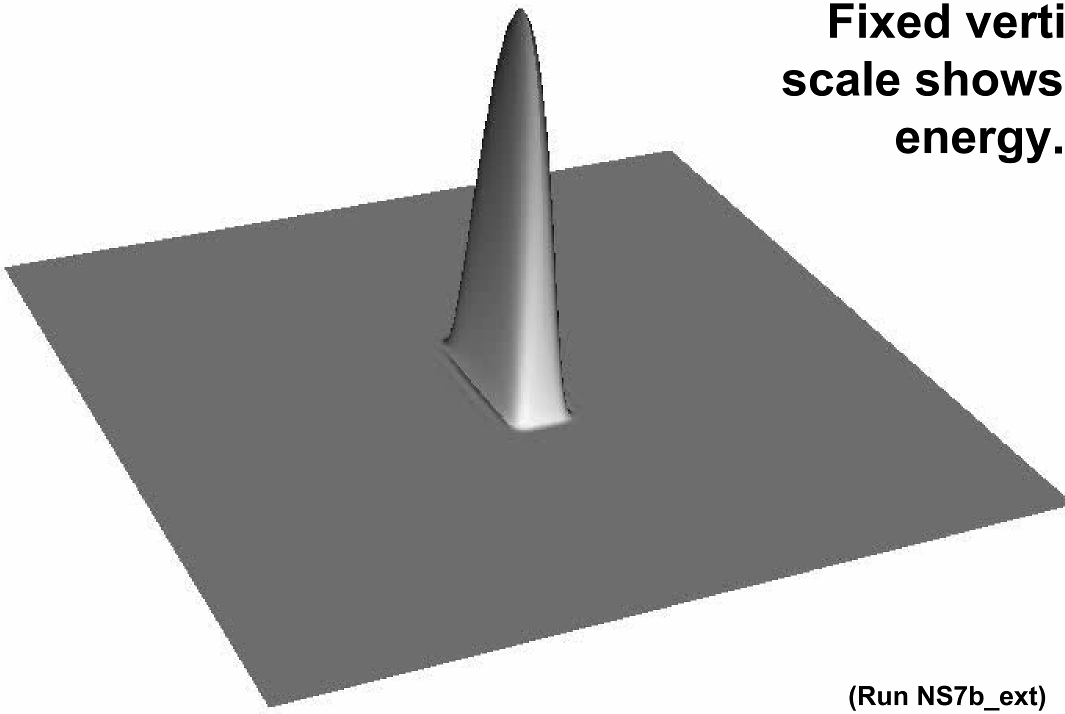
Error due to parabolic effects can be controlled: avoid scale length match between LOBC and dissipation

- To limit error associated with parabolic physics, avoid scale length match between LOBC buffer region and dissipative scale length.
- Relevant parameters include
  - *k* , coefficient of thermal conduction
  - L_{ext}, exterior domain size
  - t_{keep}, time source terms are kept ($t_{keep} = \frac{L_{ext}}{c}$)
  - L_k, thermal conduction length ($L_k = \sqrt{k t_{keep}}$)
  - ε, the error at *t*=3 (the max. variation of *e* in the domain normalized by e_0)

k	L_k	L_{ext}	t_{keep}	ε
0.5	0.44	0.5	0.38	1.2e-2
5	1.37	”	”	5.1e-3
50	4.4	”	”	8.0e-4

LOBC is effective for Navier-Stokes

- Asymmetric initial condition → oblique boundary dynamics
- Significant viscosity (Re=100) and heat conduction (Pr=1)



Fixed vertical scale shows total energy.

(Run NS7b_ext)

Future work

- Extend LOBC to dissipative MHD
- Test with dissipative MHD problem (e.g., FRC translation)

Dutch scientist Christiaan Huygens (1629-1695)