

Hybrid Kinetic-MHD Simulations at the PSI Center

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The Outline

- summary of theory FLR effects on tearing modes (V. Svidzinski, PoP 2004)
- NIMROD preliminaries
- hybrid kinetic-MHD
- initial simulation results
- stepping backwards to a slab
- stepping forward - plans



Summary of Theoretic Prediction of V. Svidzinski^a

- predicts that in the limit of $v_{\parallel} = 0$, large FLR orbits stabilize tearing modes
- treats the energetic particles as a perturbation that modifies tearing layer problem in RFP
 - backs out a conductivity tensor from linearized Vlasov equation
 - uses conductivity tensor to obtain \mathbf{J}_{hot}
 - uses Ampere's law to obtain energetic particle corrections to magnetic field and growth rate in linearized MHD equations
- examines impact of localized energetic distribution
- conjectures that finite v_{\parallel} dilutes stabilization

^aV. A. Svidzinski and S. C. Prager, "Effects of particles with large gyroradii on resistive magnetohydrodynamic stability", PoP **11** 980, 2004



NIMROD Preliminaries

- NIMROD is an initial value 3D XMHD code
- uses finite elements in two dimension, Fourier in the third
 - allows geometric flexibility
 - can handle extreme anisotropies, $\frac{\chi_{\parallel}}{\chi_{\perp}} \gg 10^6$
- semi-implicit advance, not restricted by magnetosonic CFL condition
 - model experiment relevant parameters, $S \sim 10^{7-8}$
- allows both linear and nonlinear simulations



NIMROD

- NIMROD's extended MHD equations

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} + \kappa_{divb} \nabla \nabla \cdot \mathbf{B}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\mathbf{E} = -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B}$$

$$+ \frac{m_e}{ne^2} \left[\sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} (\nabla p_{\alpha} + \nabla \cdot \Pi_{\alpha}) \right] + \frac{m_e}{ne^2} \left[\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot (\mathbf{J}\mathbf{U} + \mathbf{U}\mathbf{J}) \right]$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{U}) = \nabla \cdot D\nabla n$$

$$mn \left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \rho \nu \nabla \mathbf{V} - \nabla \cdot \Pi - \nabla \cdot p_h$$

$$\frac{n_{\alpha}}{\Gamma - 1} \left(\frac{\partial T_{\alpha}}{\partial t} + \mathbf{U}_{\alpha} \cdot \nabla T_{\alpha} \right) = -p_{\alpha} \nabla \cdot \mathbf{U}_{\alpha} - \nabla \cdot q_{\alpha} + Q_{\alpha} - \Pi_{\alpha} : \nabla \mathbf{U}_{\alpha}$$



Representation of NIMROD fields

- NIMROD fields use quadrilateral finite element-Fourier representation

$$\delta A(\mathbf{x}, t) = \sum_j A_{j,0}(t) \alpha_{j,0} + \sum_j \sum_n (A_{j,n}(t) \alpha_{j,n} + c.c.)$$

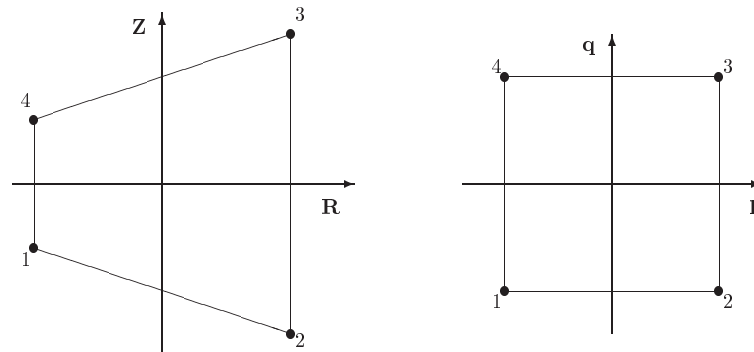
where

$$\alpha_{j,n} = N_j(p, q) \exp(in\phi)$$

(p, q) are logical coordinates, $N_j(p, q) = l_j(p)l_j(q)$ and

$$l_i(x) = \prod_{i=0, i \neq j}^k \frac{x - x_i}{x_j - x_i}$$

$k = pd + 1$, pd is the polynomial order of the Lagrange polynomial



Summary of the δf PIC Method^{abc}

- PIC is a Lagrangian simulation of phase space $f(\mathbf{x}, \mathbf{v}) \rightarrow f(\mathbf{x}(t), \mathbf{v}(t))$
 - discretize $f(\mathbf{x}, \mathbf{v})$ - sample with particles
 - equations of motion are used to advance particles
 - PIC uses spatial grid for field information
- in principle, $f(\mathbf{x}, \mathbf{v})$ contains everything
- **BUT** limited statistics - PIC is noisy, can't beat $1/\sqrt{N}$
- δf PIC **reduces the discrete particle noise** associated with conventional PIC

^aS. E. Parker and W. W. Lee, 'A fully nonlinear characteristic method for gyro-kinetic simulation', *Physics of Fluids B*, **5**, 1993

^bG. Hu and J. A. Krommes, "Generalized weighting scheme for δf particle simulation method", *Physics of Plasmas*, **1**, 1994

^cA. Y. Aydemir, "A unified MC interpretation of particle simulations...", *Physics of Plasmas*, **1**, 1994



The δf PIC Method cont.

- begin with the Vlasov Equation

$$\frac{\partial f(\mathbf{z})}{\partial t} + \dot{\mathbf{z}} \cdot \frac{\partial f(\mathbf{z})}{\partial \mathbf{z}} = 0$$

- **split distribution function** into steady state and evolving perturbation:

- $f = f_{eq}(\mathbf{z}) + \delta f(\mathbf{z}, t)$

- moments of f_{eq} easy to compute

- δf evolves along the **characteristics** $\dot{\mathbf{z}}$ i.e. the equations of motion

$$\dot{\delta f} = -\dot{\mathbf{z}} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}}$$

where $\dot{\mathbf{z}} = \dot{\mathbf{z}}_{eq} + \ddot{\mathbf{z}}$

- must choose a stationary f_{eq} ($\dot{\mathbf{z}}_{eq} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}} = 0$), often chosen as a function of canonical variables

δf and the Lorentz Equations

- Lorentz equation of motion

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B})\end{aligned}$$

- for full kinetic equations use^a

$$f_0 = f(\mathbf{x}, v^2) + \frac{1}{\omega_c} (\mathbf{v} \cdot \mathbf{b} \times \nabla f)$$

- weight equation is

$$\dot{\delta f} = -\frac{\delta \mathbf{E} + \mathbf{v} \times \delta \mathbf{B}}{B} \cdot \mathbf{b} \times \nabla f - \frac{2q}{m} \delta \mathbf{E} \cdot \mathbf{v} \frac{\partial f}{\partial v^2}$$

- reproduces drift kinetic results for energetic particle effect on (1, 1) internal kink mode
- compute perturbed energetic pressure moment of δf particles

^aM. N. Rosenbluth and N. Rostoker “Theoretical Structure of Plasma Equations”, Physics of Fluids **2** 23 (1959)

Simulation Details

- Boris push with orbit averaging to accomodate disparate time scales
- energetic ion density profile $\propto \exp \left[- \left(\frac{r}{0.45a} \right)^2 \right]$
- use hybrid kinetic-MHD equations^a

$$\rho \left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = \mathbf{J} \times \mathbf{B} - \nabla \cdot \underline{\mathbf{p}}_b - \nabla \cdot \underline{\mathbf{p}}_h$$

- project velocities into (\perp , \parallel) (wrt \mathbf{B}) components

- use CGL pressure tensor $\delta \underline{\mathbf{p}}_h = \begin{pmatrix} p_{\perp} & 0 & 0 \\ 0 & p_{\perp} & 0 \\ 0 & 0 & p_{\parallel} \end{pmatrix}$

- ultimately want full pressure tensor (work in progress)

^aC.Z.Cheng, "A Kinetic MHD Model for Low Frequency Phenomena", *J. Geophys. Rev*, **96**, 1991

Linear Simulations of Tearing Modes in a RFP

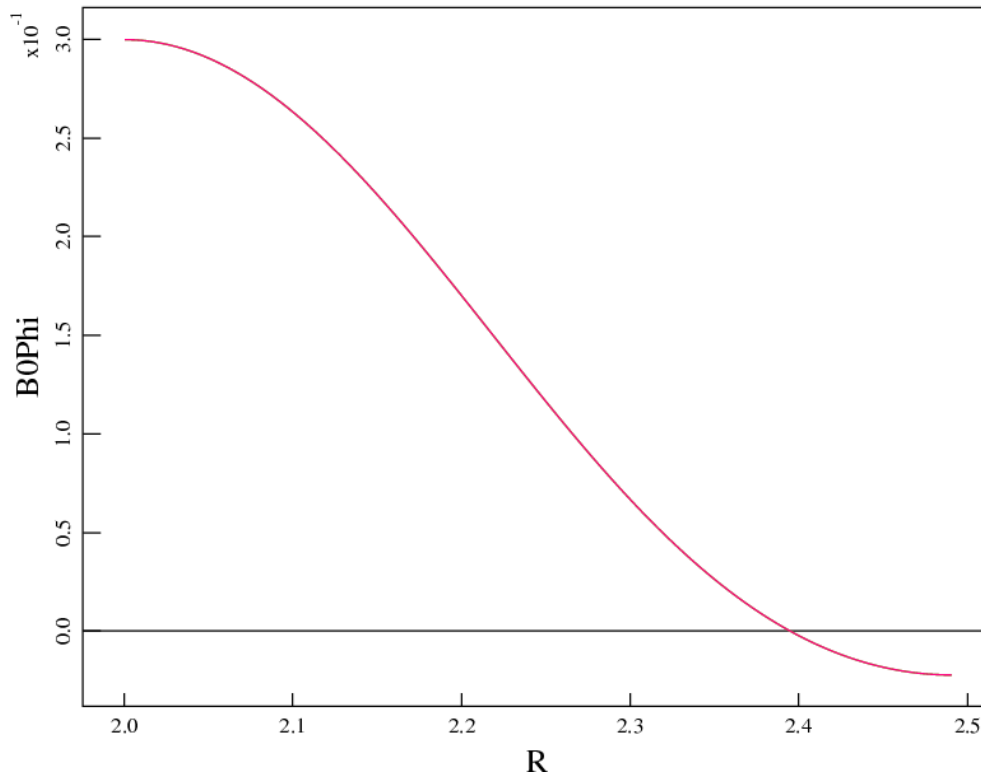
- alpha model equilibrium $\nabla \times \mathbf{B} = \mu \mathbf{B} \quad \mu = 2\Theta \left[1 - \left(\frac{r}{a} \right)^{\alpha_0} \right]$

- parameters for straight cylinder

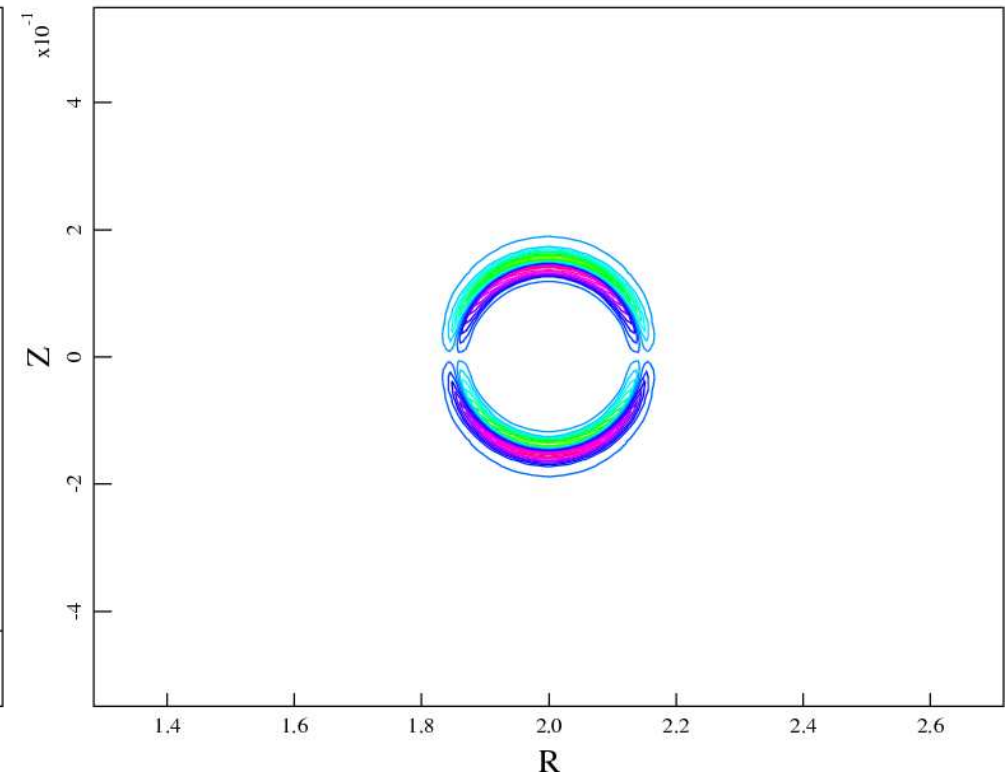
$$a = .5m, B_0 = .3T, \Theta = 1.75, \alpha_0 = 3,$$

$$S = 1.e4, ka = 2, \gamma\tau_A = 1.3e - 3$$

B0_Phi vs. R

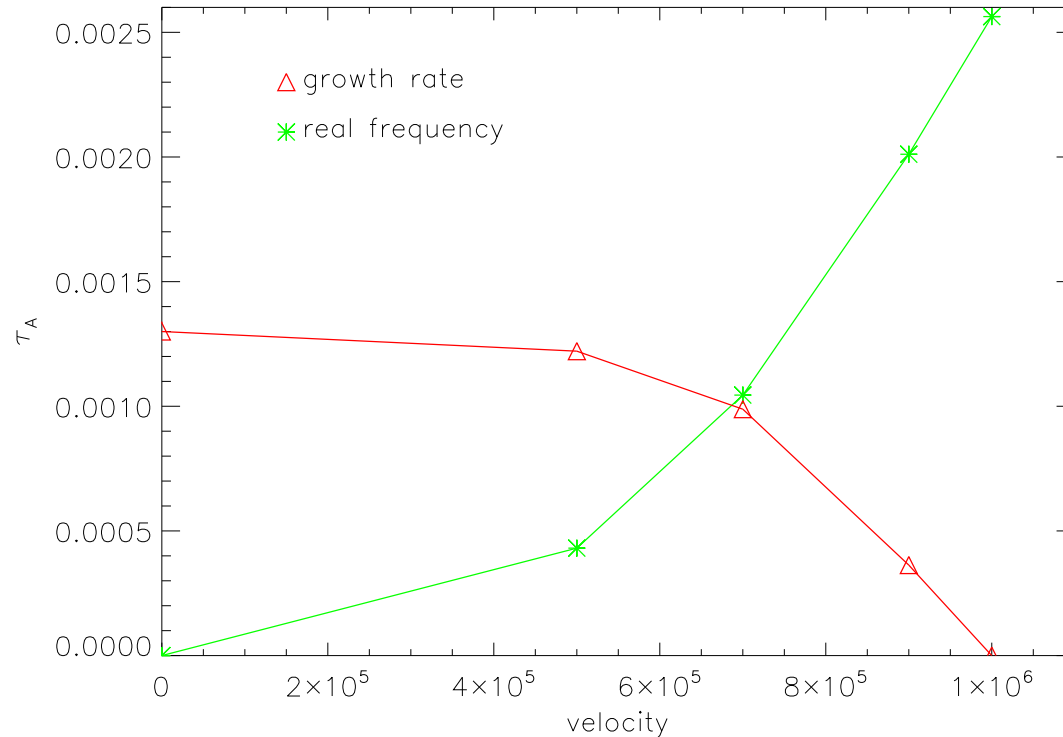


Re VPhi



FLR Stabilization of RFP Tearing Mode

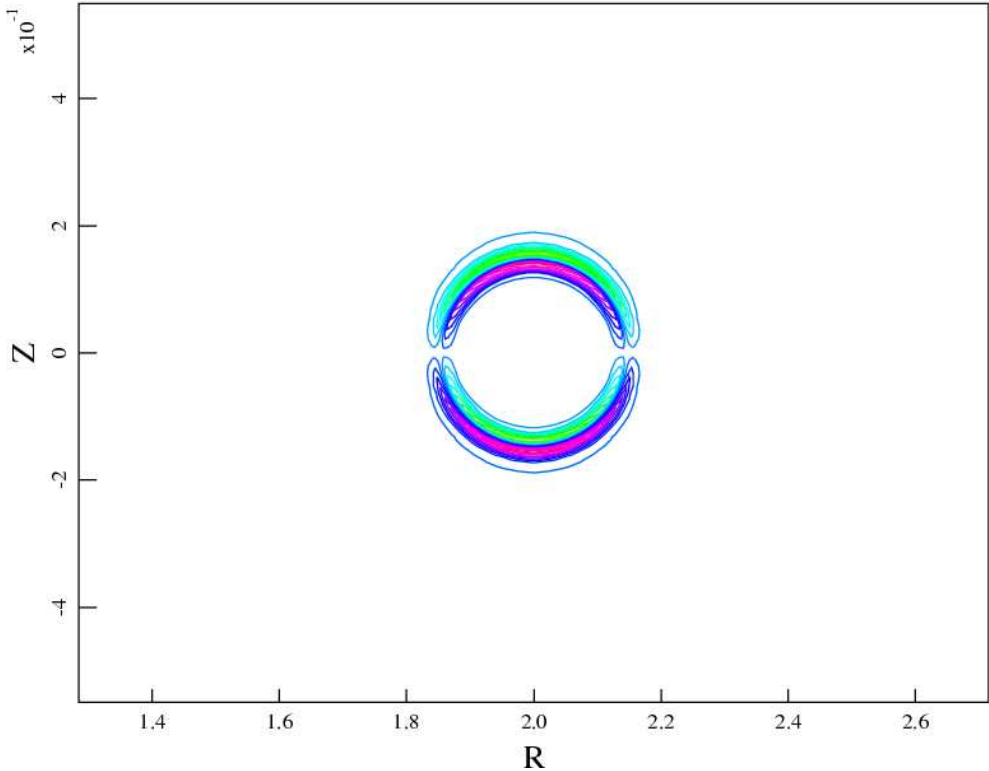
- initialize with monoenergetic particles, only $\mathbf{v} \times \delta\mathbf{B}$ in weight equation
- use **only** perpendicular pressure for comparison with theory



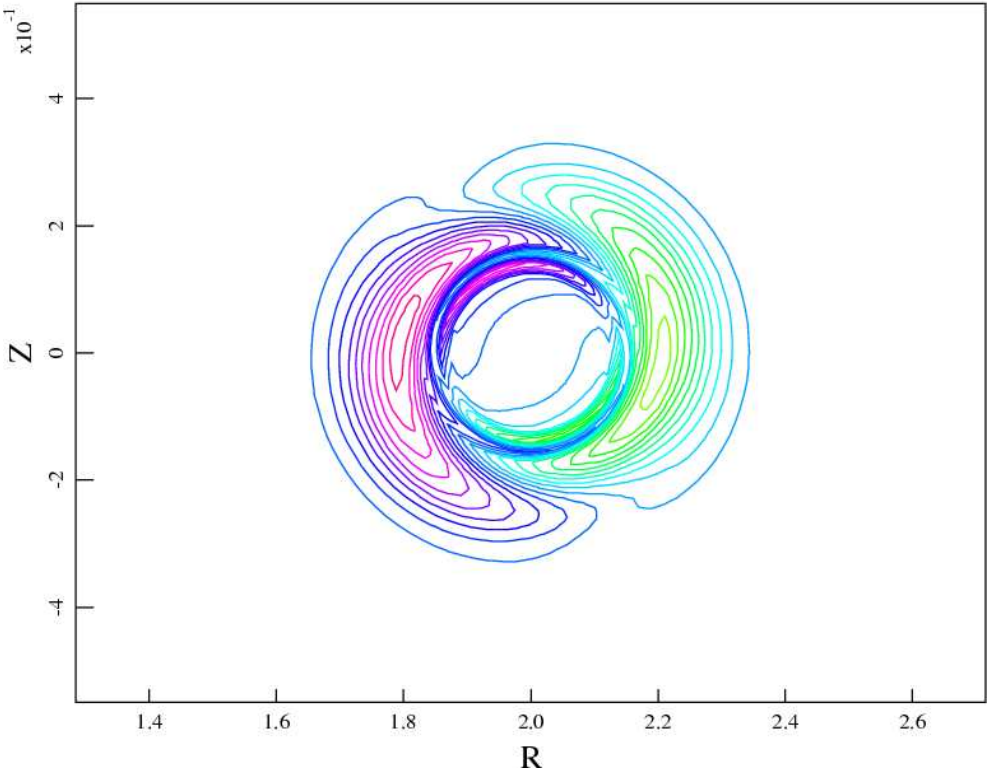
- stabilization at $\rho_h \simeq 4cm$
- simulation sees real frequency - probably due to finite spread in velocity

FLR Broadens Eigenmode Structure

Re VPhi



Re VPhi



A Step Back - Path to further development

- need improved particles for full pressure tensor
- to ease development move to rectangular mesh - slab geometry
- probably an easier problem - both physics and computation
- interesting problem in itself



Tearing mode in a slab

- strong guide field
- Gaussian current profile $J_\phi \propto \exp\left(-\frac{(x - x_0)^2}{a^2}\right)$
- two scale lengths of interest, a/ρ and $k_y\rho$
- effects of localization of energetic particles
- effect of finite v_\parallel
- distill out geometry

Stepping forward - Development plans

- move PIC-in-FEM to a CIC in nonuniform grid^a
- use polynomial shape function $S \propto \left[1 - \left(\frac{r}{R}\right)^2\right]^\alpha$ where r is distance of particle from grid node, R is radius of influence, α is parameter to be explored
- compare polynomial shape function to present implementation
- apply full pressure tensor
- move to cylindrical geometry
 - term by term comparison with V. Svidzinski
- toroidal geometry
- apply to FRCs
- implement a full f PIC

^aG. B. Jacobs and J. S. Hesthaven, 'High-order nodal discontinuous Galerkin PIC method on unstructured grids', *JCP*, **214**, 2006