

Preliminary Simulations of FLR effects on RFP tearing modes.

Charlson C. Kim
and the NIMROD Team

Plasma Science and Innovation Center
University of Washington, Seattle



Motivation and Background

- develop hybrid kinetic-MHD model with full orbit kinetics
- necessary to model ICC devices, e.g. spheromaks, RFP's, FRC's
- extensive efforts past and present - Barnes&Milroy, Park et al., Belova, Todo
- leverage successful extended MHD capabilities of NIMROD ^{a b}
- extend drift kinetic hybrid work
- test full kinetic model in stabilization of RFP tearing mode - V. Svidzinski PoP 2004
- demonstrate capabilities of simulation
- preliminary steps towards ultimate simulation

^aC. R. Sovinec, et al., “Nonlinear magnetohydrodynamic simulations using higher-order finite elements”, JCP **195**, 355 (2004)

^bD. D. Schnack, et al., “Computational modeling of fully ionized magnetized plasmas using the fluid approximation”, PoP **13**, 058103 (2006)



Outline of Talk

- model equations
 - Extended MHD fluid equations
 - δf and the kinetic equations
 - hybrid kinetic-MHD equation
- comparison to drift kinetic results
- RFP model and summary of Svidzinski's theoretic results
- simulation results
- conclusion and future directions

NIMROD

- NIMROD evolves the **extended** MHD equations

$$\begin{aligned}\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times \mathbf{E} + \kappa_{divb} \nabla \nabla \cdot \mathbf{B} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \\ \mathbf{E} &= -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B}\end{aligned}$$

$$+ \frac{m_e}{ne^2} \left[\sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} (\nabla p_{\alpha} + \nabla \cdot \Pi_{\alpha}) \right] + \frac{m_e}{ne^2} \left[\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot (\mathbf{J}\mathbf{U} + \mathbf{U}\mathbf{J}) \right]$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{U}) = \nabla \cdot D\nabla n$$

$$mn \left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \rho \nu \nabla \mathbf{V} - \nabla \cdot \Pi - \nabla \cdot p_h$$

$$\frac{n_{\alpha}}{\Gamma - 1} \left(\frac{\partial T_{\alpha}}{\partial t} + \mathbf{U}_{\alpha} \cdot \nabla T_{\alpha} \right) = -p_{\alpha} \nabla \cdot \mathbf{U}_{\alpha} - \nabla \cdot q_{\alpha} + Q_{\alpha} - \Pi_{\alpha} : \nabla \mathbf{U}_{\alpha}$$

- finite element in one plane, pseudo-spectral in 3rd direction



Summary of the δf PIC Method^{a b}

- PIC is a Lagrangian simulation of phase space $f(\mathbf{x}, \mathbf{v}) \rightarrow f(\mathbf{x}(t), \mathbf{v}(t))$
 - discretize $f(\mathbf{x}, \mathbf{v})$ - sample with ‘particles’ or markers
 - equations of motion are used to advance ‘particles’
 - spatial grid is used to compute moments of f
- in principle, $f(\mathbf{x}, \mathbf{v})$ contains everything
- PIC is noisy, can’t beat $1/\sqrt{N}$
- δf PIC **reduces the discrete particle noise** associated with conventional PIC

^aS. E. Parker and W. W. Lee, ‘A fully nonlinear characteristic method for gyro-kinetic simulation’, *Physics of Fluids B*, **5**, 1993

^bG. Hu and J. A. Krommes, ”Generalized weighting scheme for δf particle simulation method”, *Physics of Plasmas*, **1**, 1994



The δf PIC Method cont.

- begin with the Vlasov Equation

$$\frac{\partial f(\mathbf{z})}{\partial t} + \dot{\mathbf{z}} \cdot \frac{\partial f(\mathbf{z})}{\partial \mathbf{z}} = 0$$

- **split phase space** distribution into steady state and evolving perturbation:
 - $f = f_{eq}(\mathbf{z}) + \delta f(\mathbf{z}, t)$
 - moments of f_{eq} easy to compute
- δf evolves along the **characteristics $\dot{\mathbf{z}}$** (control variates MC^a)

$$\dot{\delta f} = -\tilde{\mathbf{z}} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}}$$

using $\mathbf{z} = \mathbf{z}_{eq} + \tilde{\mathbf{z}}$ and $\dot{\mathbf{z}}_{eq} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}} = 0$

- must choose a stationary f_{eq} , often chosen as a function of canonical variables

^aA. Y. Aydemir, “A unified MC interpretation of particle simulations...”, *Physics of Plasmas*, **1**, 1994

δf and the Lorentz Equations

- Lorentz equation of motion

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B})\end{aligned}$$

- for full kinetic equations use^a

$$f_0 = f(\mathbf{x}, v^2) + \frac{1}{\omega_c} (\mathbf{v} \cdot \mathbf{b} \times \nabla f)$$

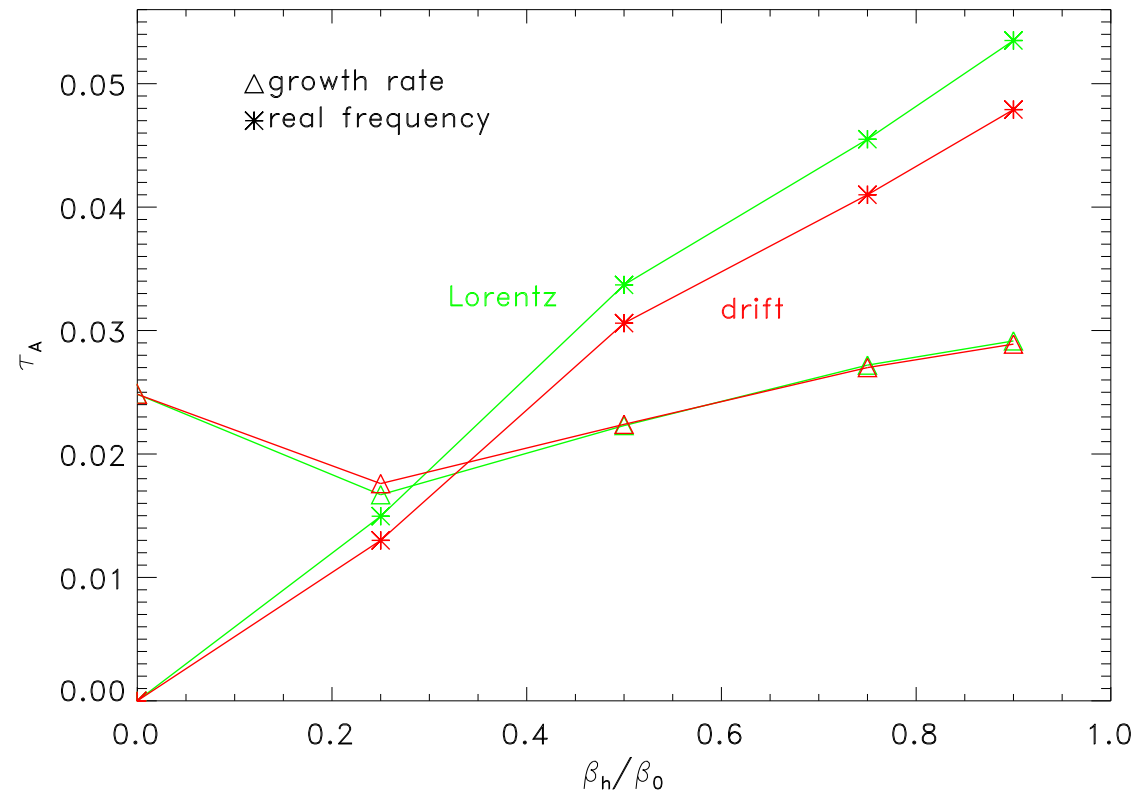
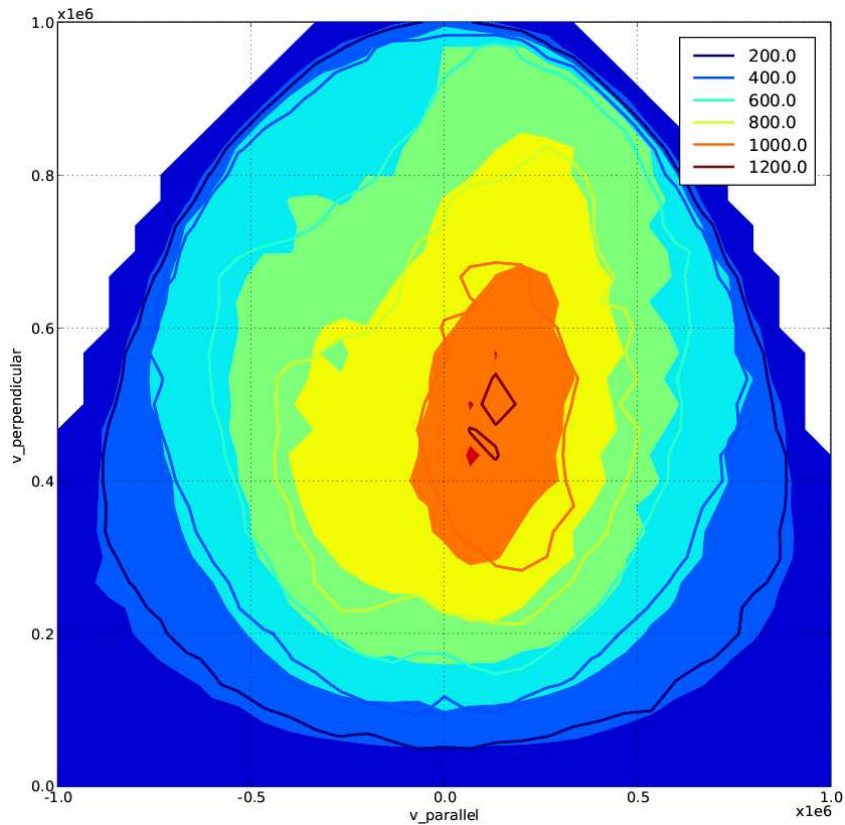
- weight equation is

$$\dot{\delta f} = -\frac{\delta \mathbf{E} + \mathbf{v} \times \delta \mathbf{B}}{B} \cdot \mathbf{b} \times \nabla f - \frac{2q}{m} \delta \mathbf{E} \cdot \mathbf{v} \frac{\partial f}{\partial v^2}$$

- satisfying in that it is very similar to drift kinetic weight equation

^aM. N. Rosenbluth and N. Rostoker “Theoretical Structure of Plasma Equations”, Physics of Fluids **2** 23 (1959)

Boris Algorithm Nicely Recovers Drift Kinetic Results



- velocity space overlap - infer drift trajectories are recovered by Lorentz push
- reproduce linear (1, 1) kink mode stabilization by hot particles

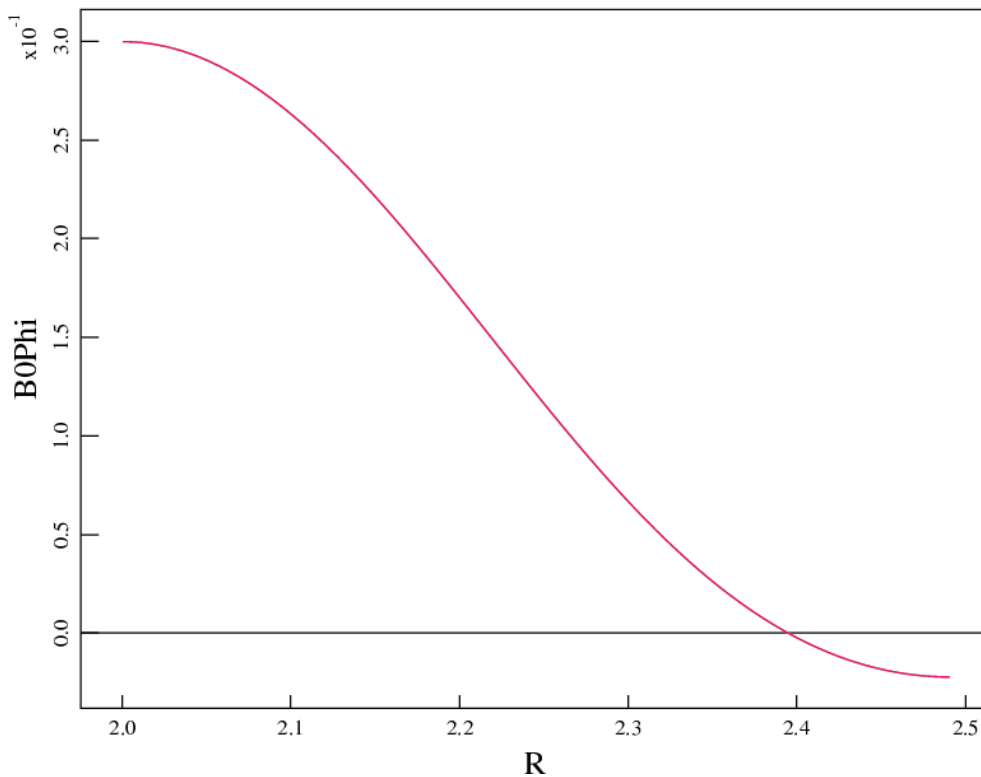
Linear Simulations of Tearing Modes in a RFP

- alpha model equilibrium $\nabla \times \mathbf{B} = \mu \mathbf{B}$ $\mu = 2\Theta \left[1 - \left(\frac{r}{a} \right)^{\alpha_0} \right]$
- parameters for straight cylinder

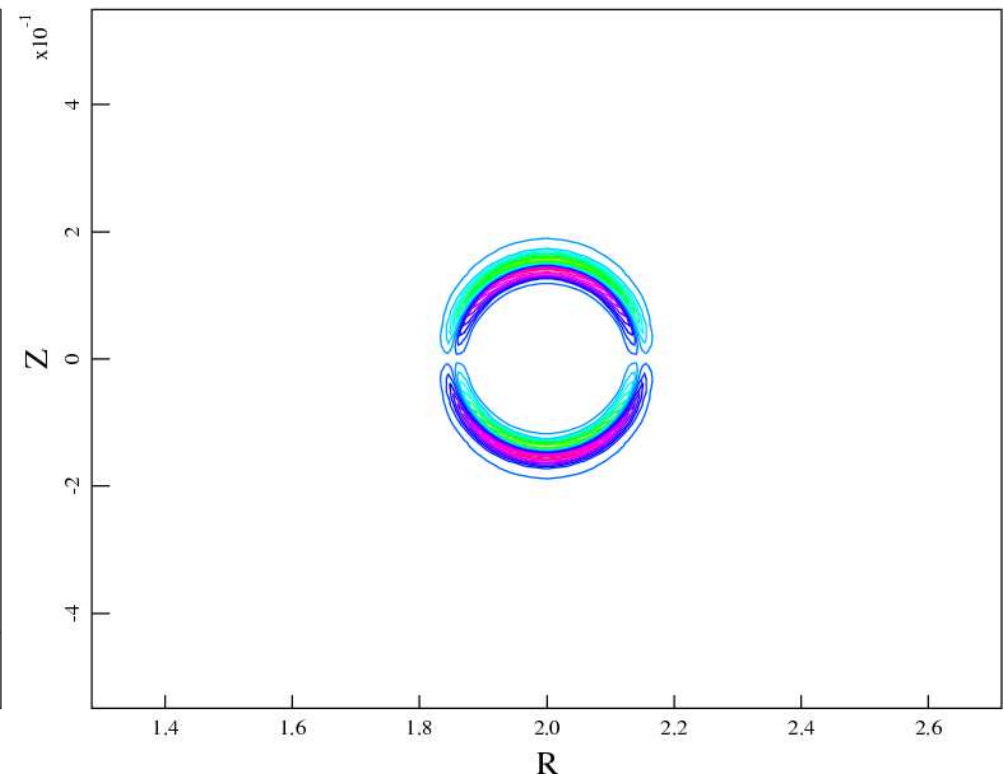
$$a = .5m, B_0 = .3T, \Theta = 1.75, \alpha_0 = 3,$$

$$S = 1.e4, ka = 2, \gamma\tau_A = 1.3e - 3$$

B0_Phi vs. R



Re VPhi



Summary of Theoretic Prediction of V. Svidzinski^a

- calculates dielectric response of energetic particles for \mathbf{E} , \mathbf{B}
- uses conductivity tensor $\sigma^{\mathbf{E}}, \sigma^{\mathbf{B}}$ to calculate \mathbf{J}_h contribution from energetic particles
- focuses on $\sigma_{yy}^{\mathbf{E}}$ and $\sigma_{xy}^{\mathbf{B}}$
- uses Ampere's relation $\nabla \times \mathbf{B} = \mathbf{J} + \mathbf{J}_h$
- treats the energetic particles as a perturbation to $\mathbf{B} = \tilde{\mathbf{B}} + \mathbf{B}_h$ to solve the tearing layer problem in RFP
- uses $f(\mathbf{v}) \propto \delta(v_{\perp} - v_0)$
- show increasing stabilization with increasing ρ/a

^aV. A. Svidzinski and S. C. Prager, "Effects of particles with large gyroradii on resistive magnetohydrodynamic stability", PoP **11** 980, 2004



Simulation Details

- Boris push with orbit averaging to accomodates disparate time scales
- energetic ion density profile $\propto \exp \left[- \left(\frac{r}{0.45a} \right)^2 \right]$
- use hybrid kinetic-MHD equations^a

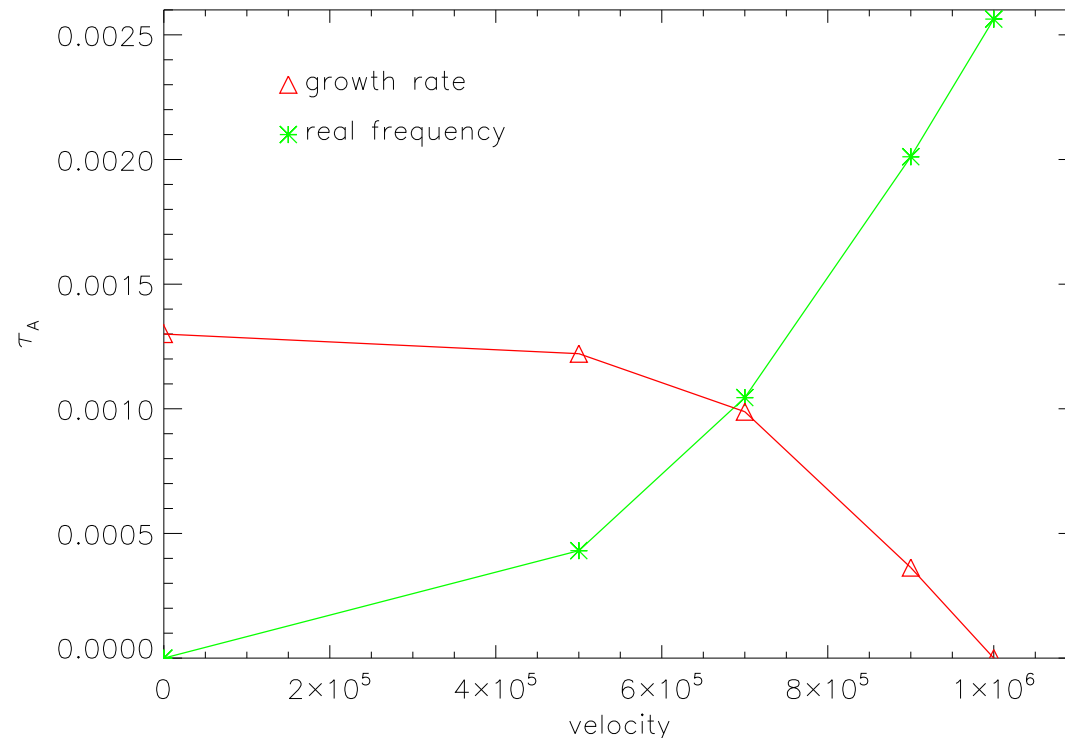
$$\rho \left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = \mathbf{J} \times \mathbf{B} - \nabla \cdot \underline{\mathbf{p}}_b - \nabla \cdot \underline{\mathbf{p}}_h$$

- use CGL pressure tensor $\delta \underline{\mathbf{p}}_h = \begin{pmatrix} p_{\perp} & 0 & 0 \\ 0 & p_{\perp} & 0 \\ 0 & 0 & p_{\parallel} \end{pmatrix}$

^aC.Z.Cheng, "A Kinetic MHD Model for Low Frequency Phenomena", *J. Geophys. Rev*, **96**, 1991

FLR Stabilization of RFP Tearing Mode

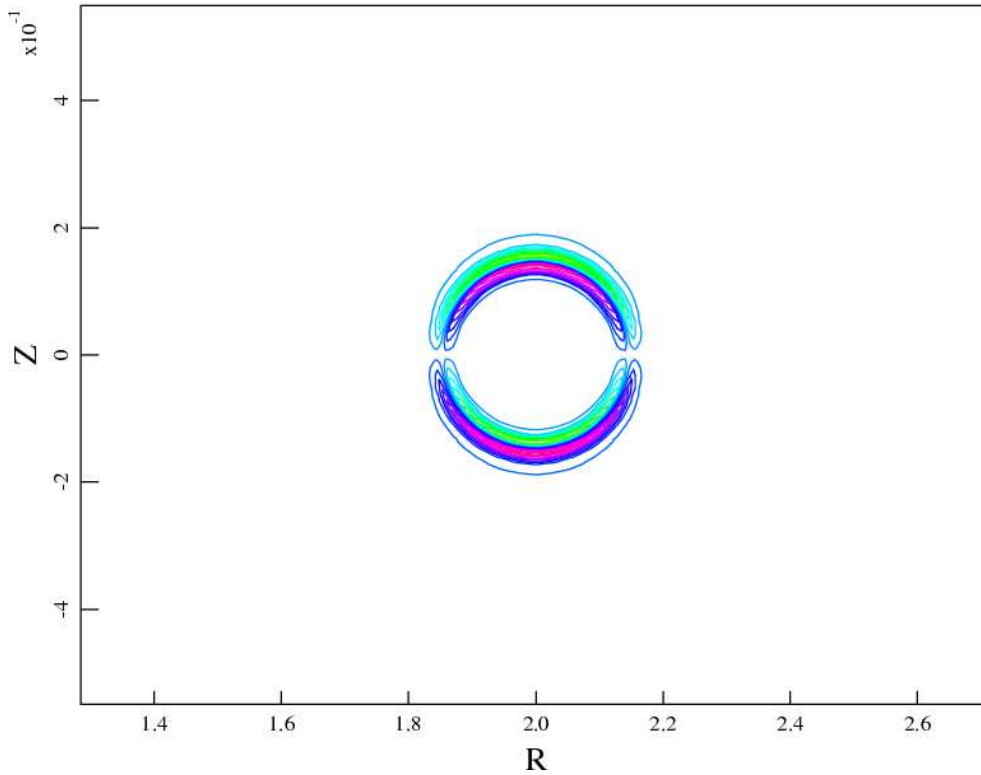
- initialize with monoenergetic particles, only $\mathbf{v} \times \delta\mathbf{B}$ in weight equation
- use **only** perpendicular pressure for comparison with theory



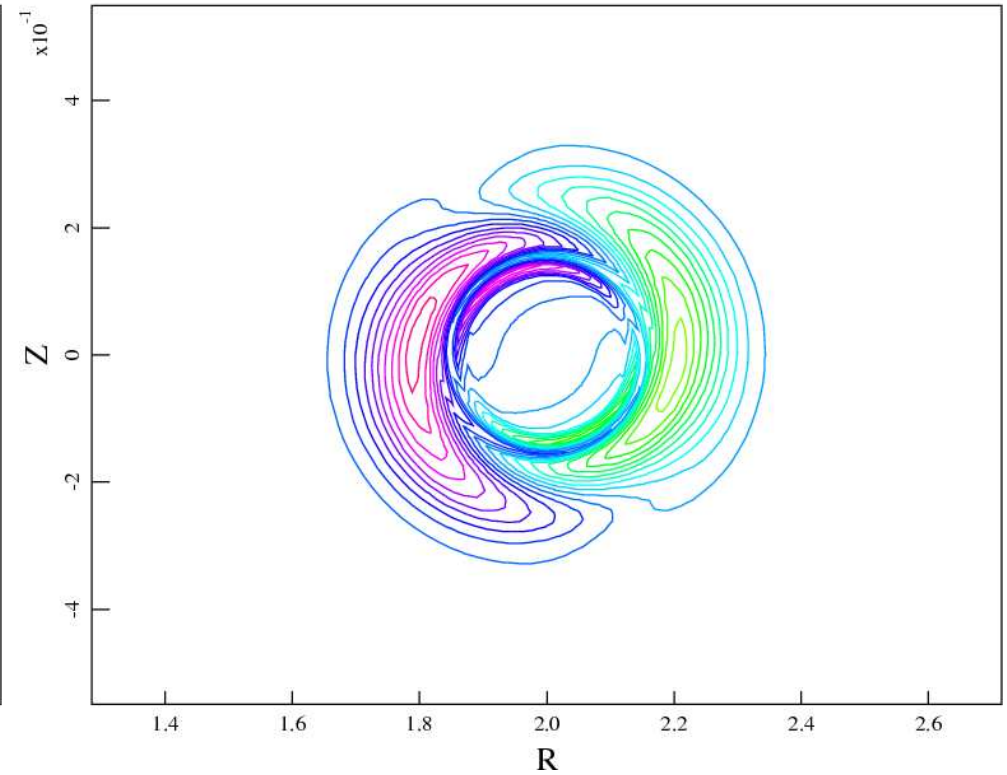
- stabilization at $\rho_h \simeq 4cm$
- simulation sees real frequency - probably due to finite spread in velocity

FLR Broadens Eigenmode Structure

Re VPhi



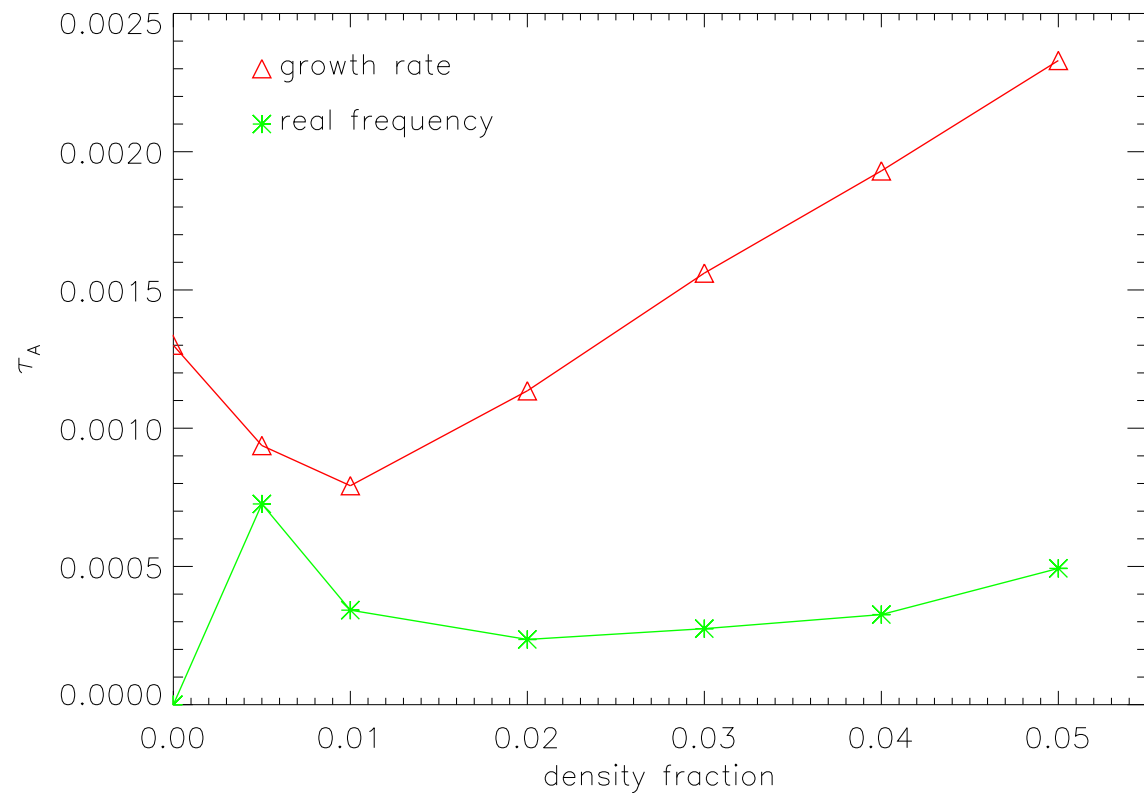
Re VPhi



- encouraging results \rightarrow turn on more physics

?!Excitation of FLR driven Instability!?

- scan in energetic density
- $\delta\mathbf{E} + \mathbf{v} \times \delta\mathbf{B}$ in weight equation
- both parallel and perpendicular pressure

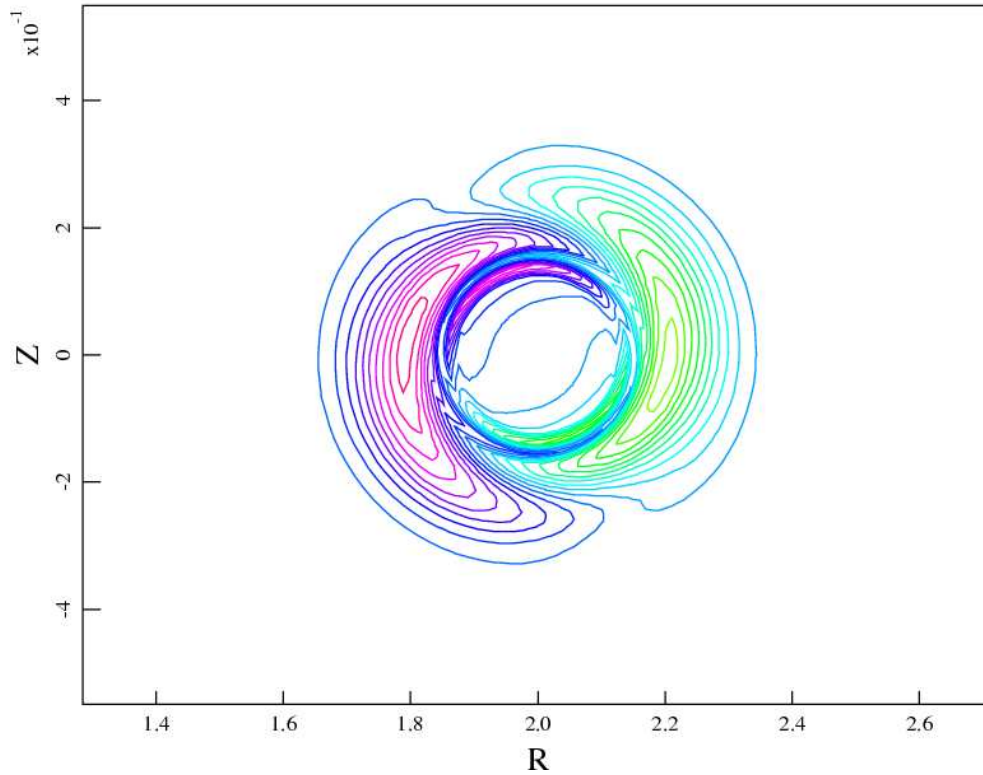


- dramatically changes “mode-ology”

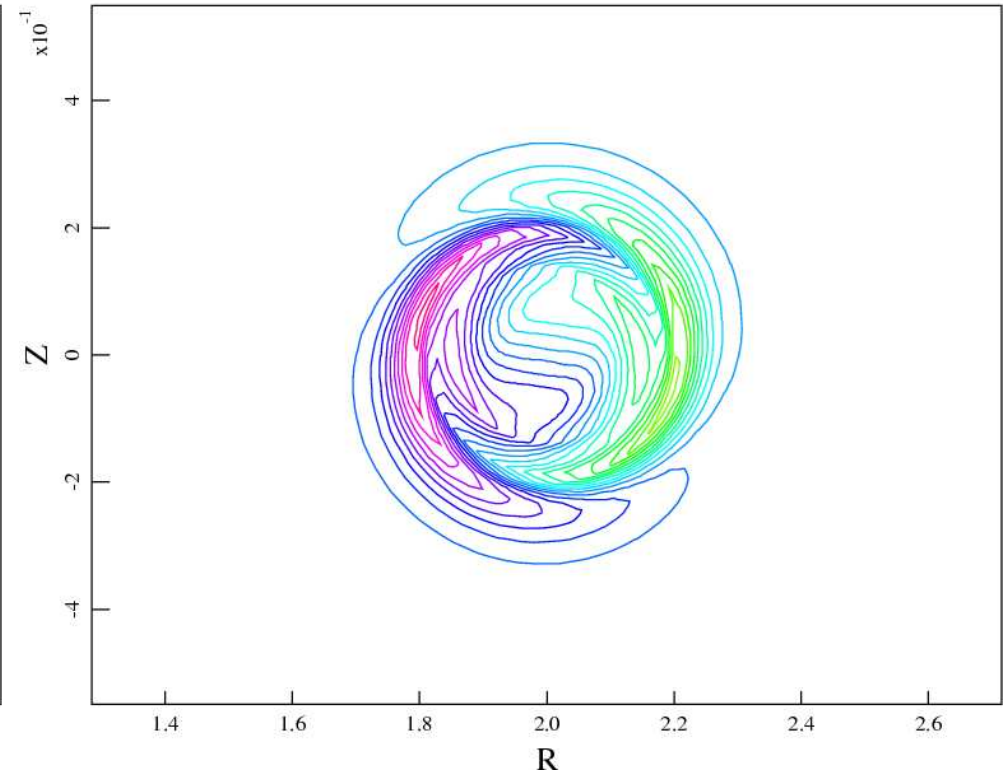


?!FLR Eigenmode!?

Re VPhi



Re VPhi



- no semblance of original tearing mode structure

End Remarks

- develop hybrid kinetic-MHD model with full orbit kinetics
- compares well to drift kinetic results
- agrees with predictions of Svidzinski's theoretic results
- hints at interesting 'mode-ology' with inclusion of more kinetic physics
- work continues on extending the model and algorithm
- stay tuned

