NIMROD simulations of flux injection in a coplanar gun

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Motivation

- inspired by successful SSPX simulations of coaxial flux injection and sustainment[a]
- coplanar flux injection is used in several ICC experiments (e.g. P. Bellan's experiment[b], Woodruff Scientific)
- study physics and numerics of helicity injection and flux amplification
- simulations will help improve operation and efficiency by elucidating physical processes of injection, columnation, reconnection, and flux amplification

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[a] Hooper, E. B., et al., "NIMROD resistive magnetohydrodynamic simulations of spheromak physics", PoP **15**, 2008

[b] Hsu and Bellan, "On Jets, Kinks, and spheromaks formed by a planar magnetized coaxial gun", PoP **12**, 2005

Summary

- axisymmetric NIMROD simulations of coplanar flux injection
    - challenging spatial scales of coplanar injector
- effects of Hall physics in flux injection
- modest success is encouraging, however still challenging spatial and temporal requirements, particularly as we progress to include more physics
    - more toroidal resolution
    - smaller dissipation parameters
    - self consistent dissipation/collision models
    - kinetics

NIMROD[a] (NonIdeal MHD with Rotation - Open Discussion)

- massively parallel 3-D MHD simulation
- finite elements in poloidal plane and Fourier modes in toroidal direction → axisymmetric geometry
- utilizes Lagrange type quadrilateral structured finite elements in 2-D
- can handle extreme anisotropies, $\frac{\chi_{\parallel}}{\chi_{\perp}} \gg 1$
- flexibility to model general geometry → real experiments
- model experiment relevant parameters, $S > 10^7$
- semi-implicit advance, not restricted by magnetosonic CFL condition
- assumes a steady state background and evolves perturbed quantities $\rightarrow A(\mathbf{x},t) = A_s(\mathbf{x}) + \delta A(\mathbf{x},t)$
- allows linear and nonlinear simulations

NIMROD equations

- NIMROD evolves the extended MHD equations

$$
\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times \mathbf{E} \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \\
\mathbf{E} &= -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} \\
&\quad + \frac{m_e}{ne^2} \left[\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot (\mathbf{JU} + \mathbf{UJ}) \right. \\
&\quad \left. + \sum_\alpha \frac{q_\alpha}{m_\alpha} (\nabla p_\alpha + \nabla \cdot \Pi_\alpha) \right] \\
\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{U}) &= \nabla \cdot D \nabla n \\
mn\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U}\right) &= \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi - \nabla \cdot p_h \\
\frac{n_\alpha}{\gamma - 1}\left(\frac{\partial T_\alpha}{\partial t} + \mathbf{U}_\alpha \cdot \nabla T_\alpha\right) &= -\nabla \cdot q_\alpha + Q_\alpha \\
&\quad - p_\alpha \nabla \cdot \mathbf{U}_\alpha - \Pi_\alpha : \nabla \mathbf{U}_\alpha
\end{aligned}
$$

Inductive Flux Injection Model

- NIMROD has two available flux injection models[a]
  - direct flux injection via Faraday's law (specifying tangential **E** field at the boundary)
  - inductive flux injection via Ampere's law (specifying a tangential **B** field at the boundary)
- inductive flux injection (i.e. specifying input current) is closer to (coaxial gun source) experiment
- exploit integral Ampere's law $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$
- specify B_ϕ at the boundary corresponding to coaxial gap to induce poloidal current in simulation domain

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[a] C. Sovinec, "Ohmic Current Drive in NIMROD Simulation", NIMROD internal note, 2005

Inductive Flux Injection Model cont.

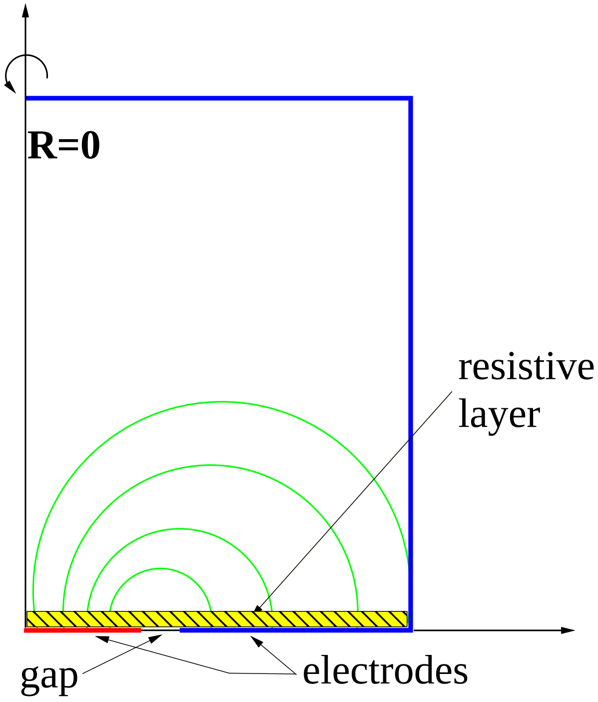
- along the boundary corresponding to flux gap specify $B_\phi R$
  - otherwise boundary condition is no-slip perfect conductor
- prescribed $B_\phi R$ induces a poloidal current in the simulation domain
- amplitude of $B_\phi R$ may vary with time (e.g. constant slope or programmed from experiment)

? thin highly resistive layer along bottom to rapidly diffuse flux across bottom

- the resulting $\mathbf{J} \times \mathbf{B}$ force pulls in the flux, drives columnation

Coplanar Flux Injection Simulation

- cylindrical vessel with small flux gap of a few centimeters

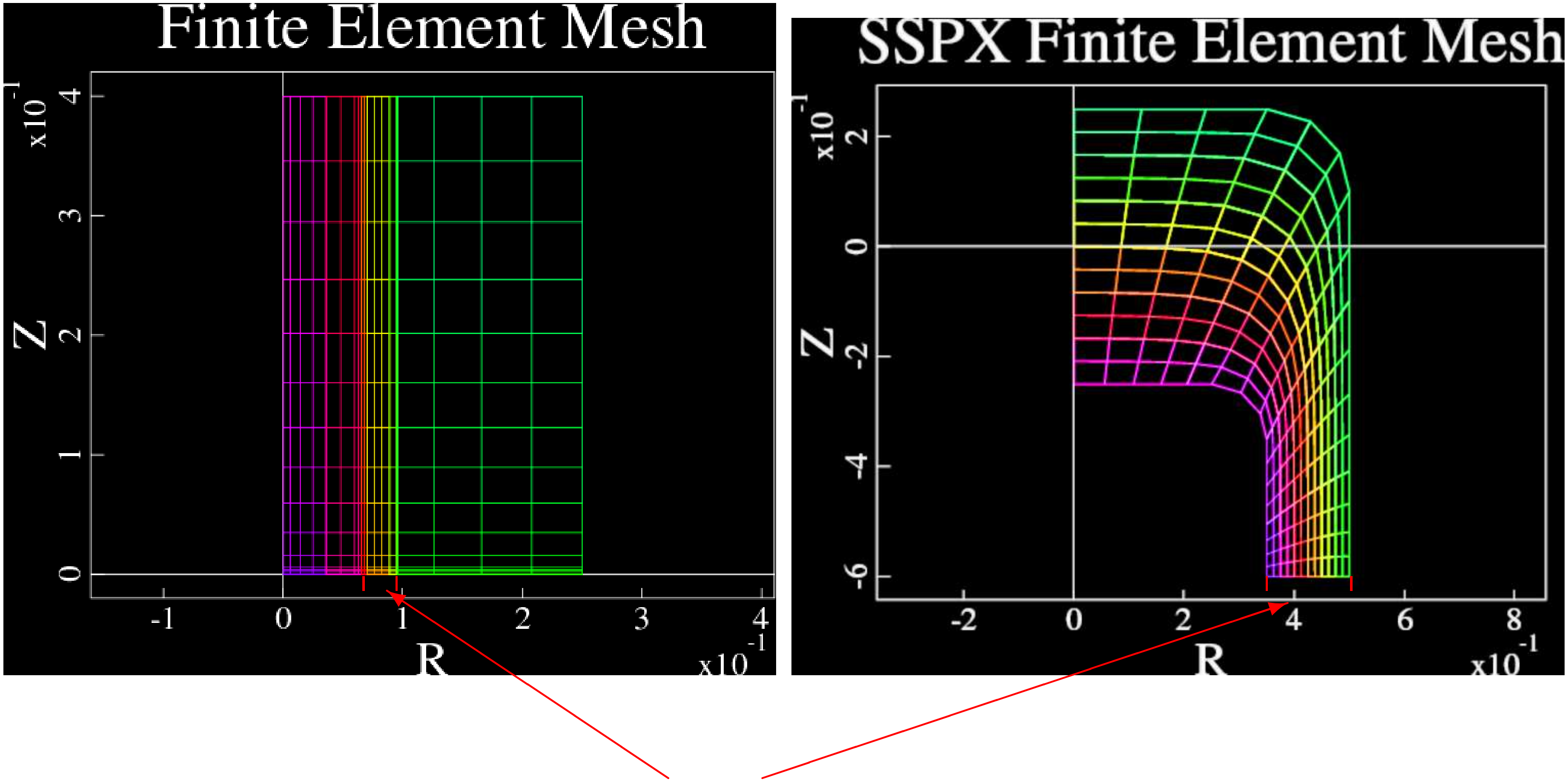


- vacuum field is dipole-like $\sim .1mWb$
  - flux gap located at top of the arc
- peak current is $40 - 100kA$ ramped over $4 - 15\mu s$
- thin highly resistive layer across bottom 10^5 larger than background resistivity

Coplanar Flux Injection is more challenging than coaxial injection

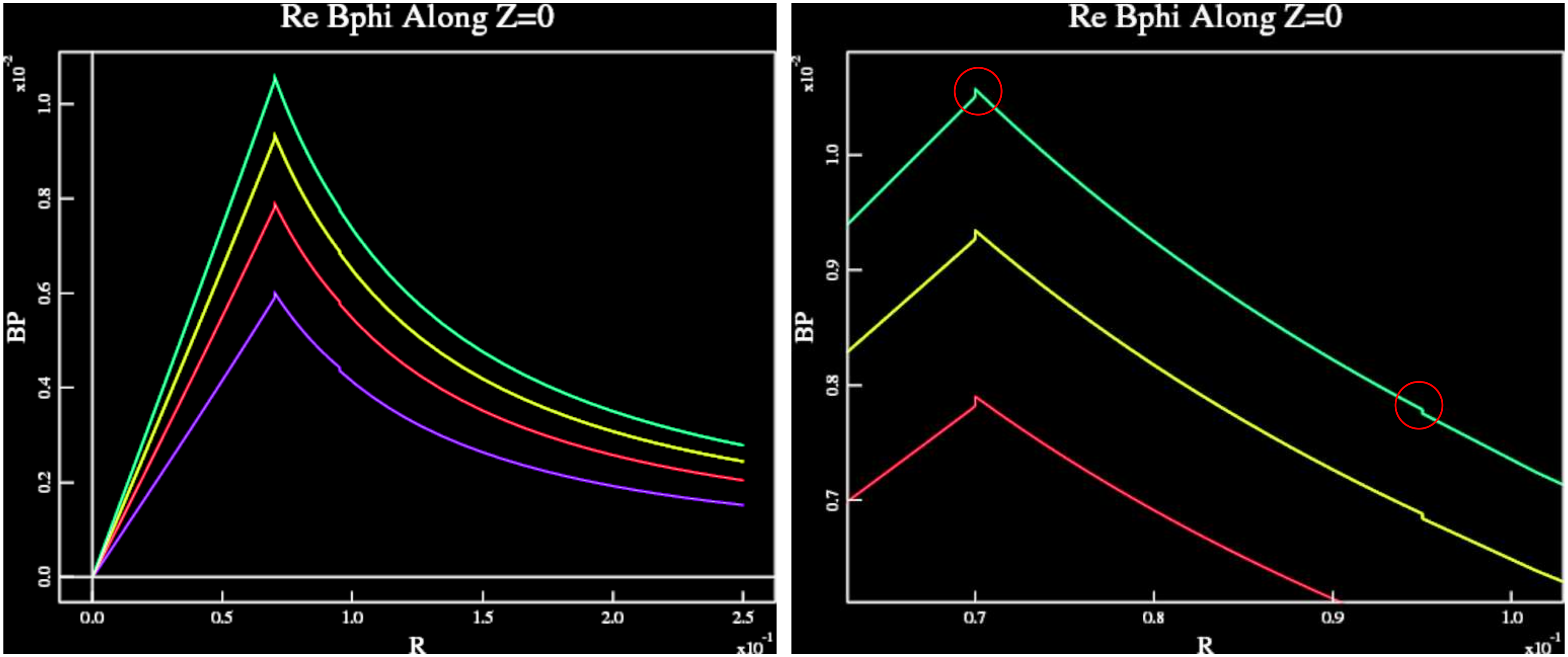
- SSPX simulations use $\sim 400kA$ ramped over $\sim 100\mu s$
  - coplanar injection simulations use $10 - 100\ kA$ ramped over $5 - 10\ \mu s$
  - initial magnetic flux is $\sim 10\times$ smaller
  - larger inductive $\mathbf{E}$ field
- coplanar geometry is more challenging than coaxial gun
  - coaxial gun is well resolved
  - no discontinuous jump in $\frac{\partial B_\phi}{\partial R}$
- $\lambda_g^{SSPX} \sim 10 - 15m^{-1}$ - we attempt $\lambda_g \sim 10 - 100m^{-1}$
- both spatially and temporally more challenging simulation

Coplanar Flux Injection is more challenging than coaxial injection cont.



flux gap

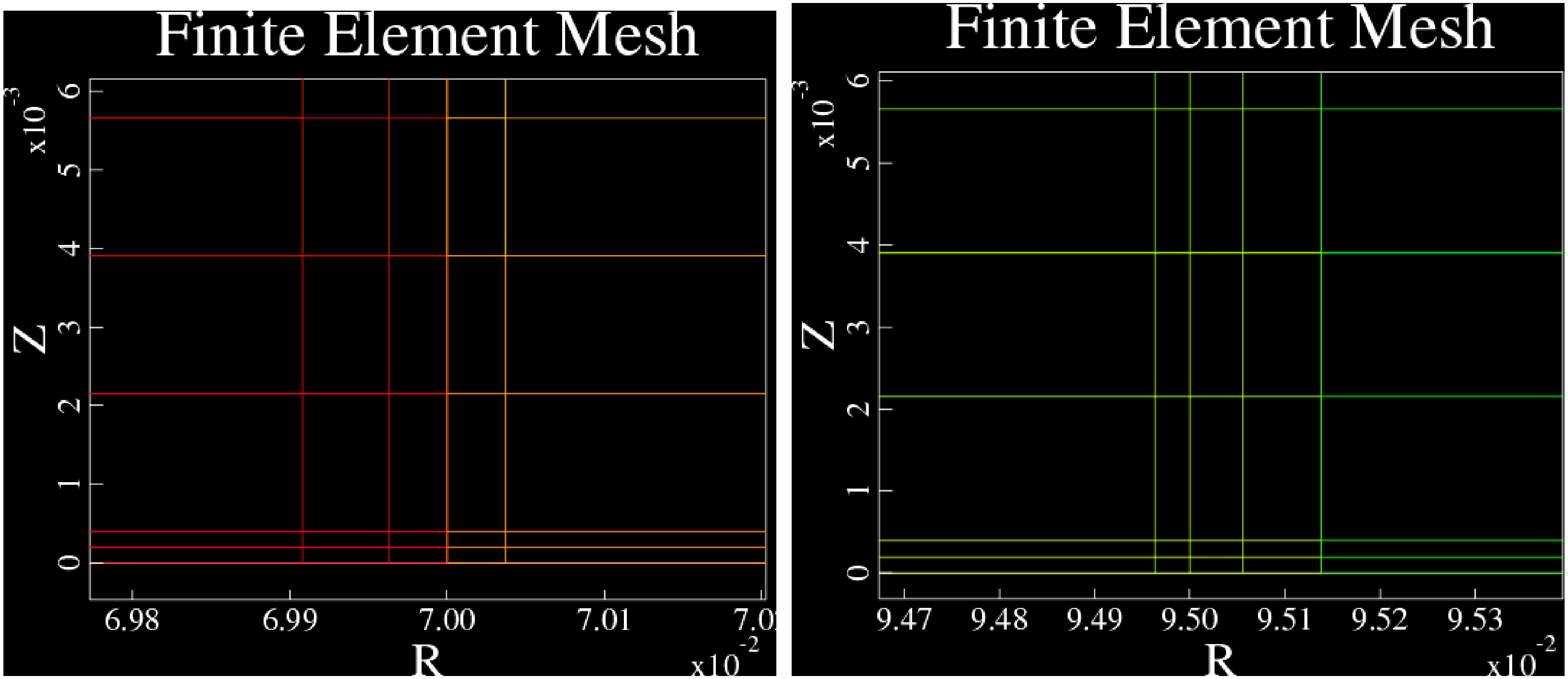
B_ϕ along Z=0 shows discontinuity



- 4 curves are at $t = [.53, .69, .82, .93]\mu s$
- note discontinuity at $R = .07$ and $.095$ where boundary condition is specified

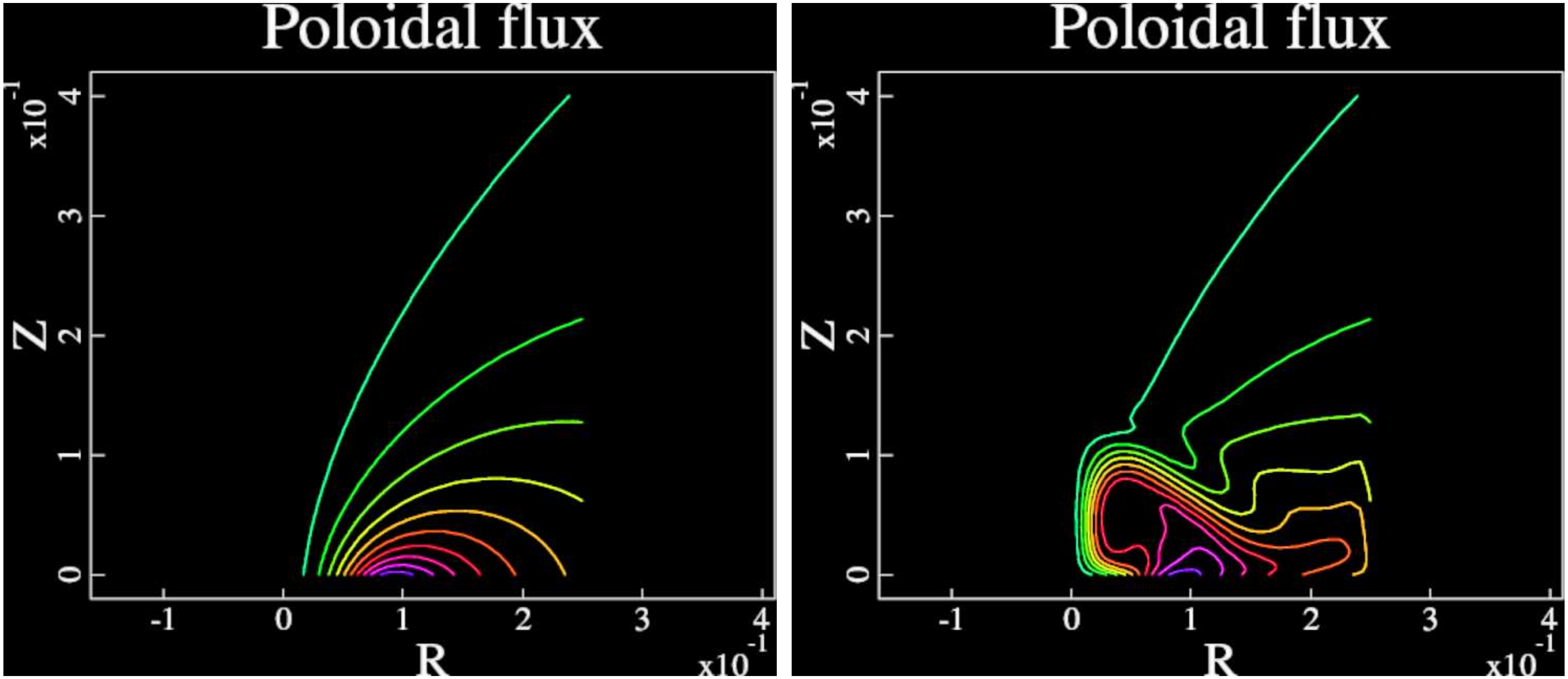
Zoomed in view of Coplanar Injection grid

discontinuous jump in $\frac{\partial B_\phi}{\partial R}$ requires high resolution



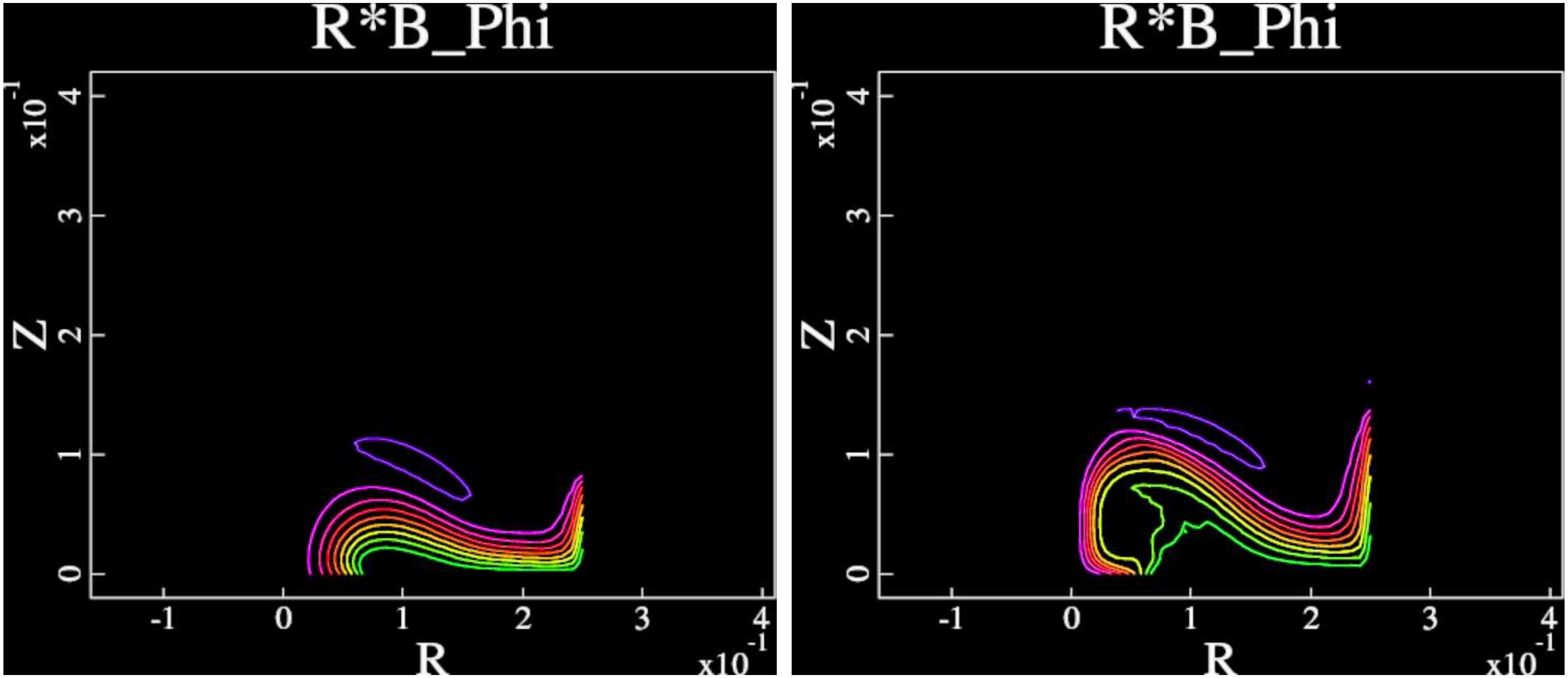
note the scale

Poloidal Flux Compression after $\sim 4.5\mu s$



- 10 contour lines $[-.13, .9]mWb$
- contour lines same value in both plots

Evolution of Poloidal Current



- left plot at $2.9\mu s$, contour range $[1.2, 11.7]kA$
- right plot at $4.2\mu s$, contour range $[1.8, 16.9]kA$
- current ramp time $10\mu s$, peak current $40kA$

Ongoing Work and Issues

- for resistive MHD fast injection results in strong Alfvenic flows
- Hall physics helps relieve strong flow issues
- but could drive smaller scale fluctuations
- still constrained by strong flows
- flux gap model is incomplete
  - no-slip perfect conductor b.c. no longer applies
  - consistent b.c. remains to be resolved