

# Extension of Drift Kinetic Hot Particles to Full Orbits in NIMROD

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## Background and Motivation

- develop hybrid kinetic-MHD model with full orbit kinetics
- necessary to model ICC devices, e.g. spheromaks, RFP's, FRC's
- extensive efforts past and present - Barnes&Milroy, Park et al, Belova, Todo
- leverage successful extended MHD capabilities of NIMROD<sup>a</sup>
- extend drift kinetic hybrid work
- also gives a point of contact and reality check
- first test of full kinetic model is stabilization of tearing RFP tearing mode - V. Svidzinski PoP 2004
- preliminary steps towards ultimate simulation

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<sup>a</sup>D. D. Schnack, et al, "Computational modeling of fully ionized magnetized plasmas using the fluid approximation", PoP **13**, 058103 (2006)



## Full Kinetic Equations

- Lorentz equation of motion

$$\dot{\mathbf{x}} = \mathbf{v} \quad \dot{\mathbf{v}} = \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (1)$$

- for full kinetic equations use<sup>a</sup>

$$f_0 = f(\mathbf{x}, v^2) + \frac{1}{\omega_c} (\mathbf{v} \cdot \mathbf{b} \times \nabla f) \quad (2)$$

- weight equation is

$$\delta \dot{f} = -\frac{\delta \mathbf{E} + \mathbf{v} \times \delta \mathbf{B}}{B} \cdot \hat{\mathbf{b}} \times \nabla f - \frac{q}{m} \delta \mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{v}} \quad (3)$$

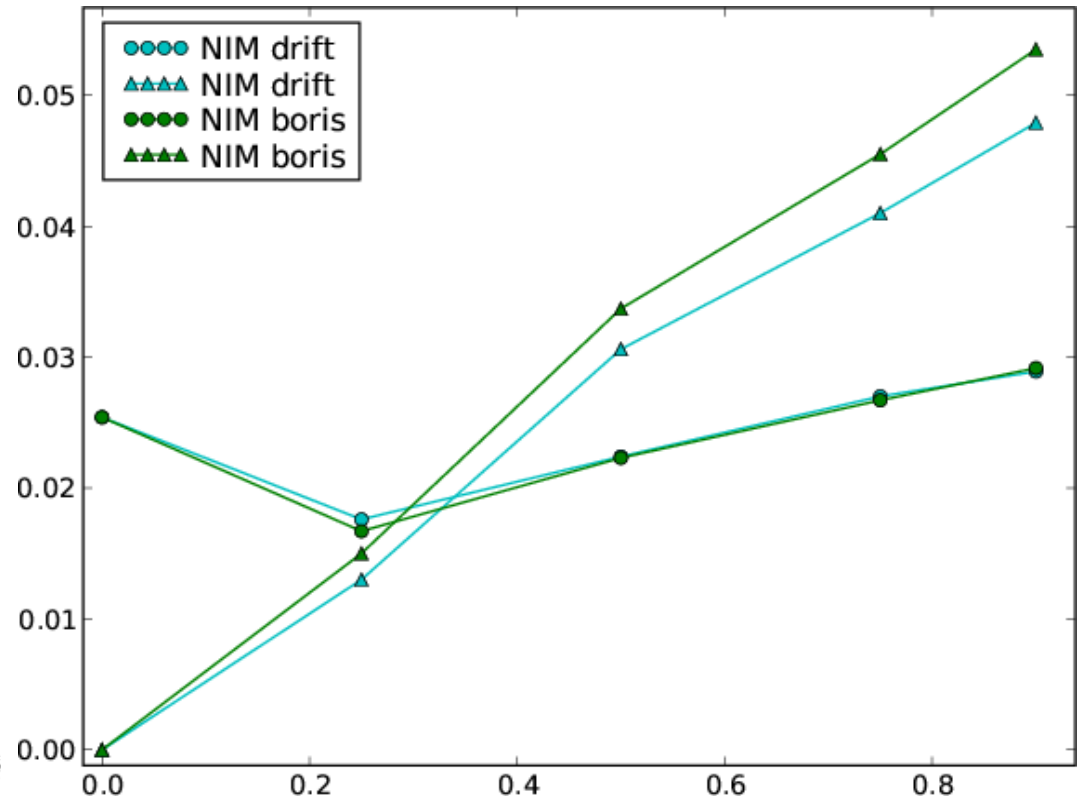
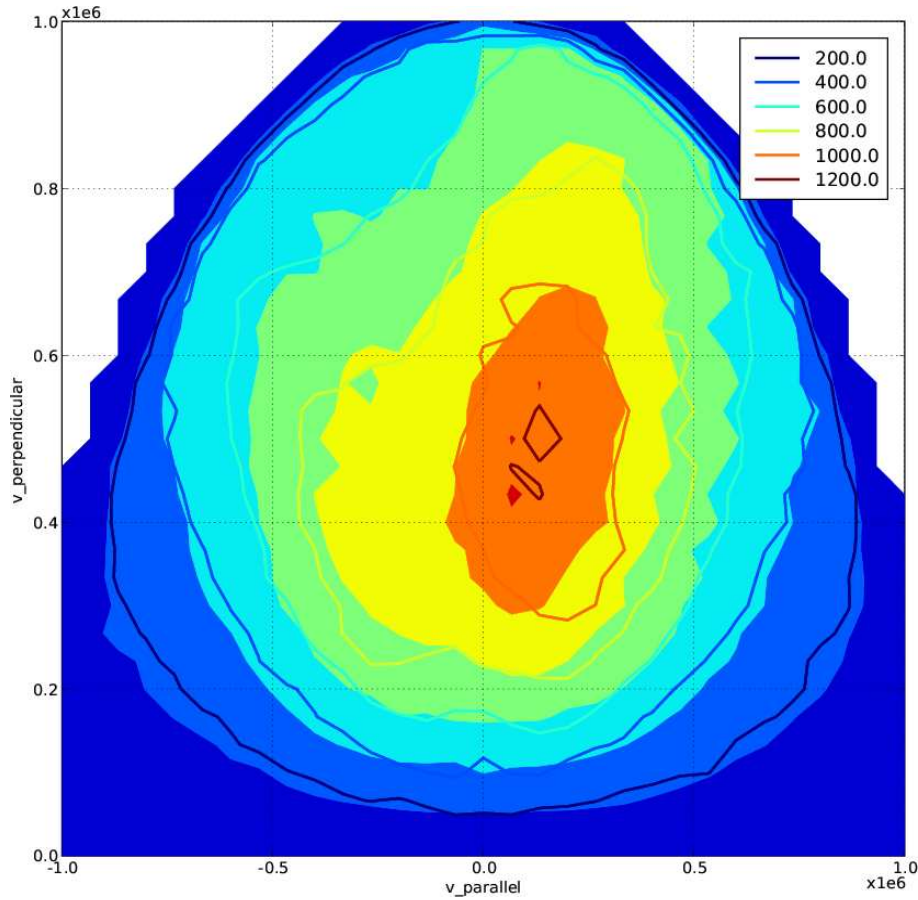
- recovers dominant terms of drift kinetic weight equation -  $\mathbf{v}_{E \times B}$  and  $v_{\parallel}$
- in the limit  $n_h \ll n_0$ ,  $\beta_h \sim \beta_0$

$$\rho \left( \frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = \mathbf{J} \times \mathbf{B} - \nabla \cdot \underline{\mathbf{p}}_b - \nabla \cdot \underline{\mathbf{p}}_h$$

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<sup>a</sup>M. N. Rosenbluth and N. Rostoker “Theoretical Structure of Plasma Equations”, Physics of Fluids vol2 23 (1959)

# Boris Algorithm nicely recovers Drift Kinetic results



- velocity space overlap - infer drift trajectories are recovered by Lorentz push
- reproduce linear (1, 1) kink mode stabilization by hot particles



## Summary of Theoretic Prediction of V. Svidzinski<sup>a</sup>

- calculates dielectric response of energetic particles for  $\mathbf{E}$ ,  $\mathbf{B}$
- uses conductivity tensor  $\sigma^{\mathbf{E}}, \sigma^{\mathbf{B}}$  to calculate  $\mathbf{J}_h$  contribution from energetic particles
- uses Ampere's relation  $\nabla \times \mathbf{B} = \mathbf{J} + \mathbf{J}_h$
- treats the energetic particles as a perturbation to  $\mathbf{B} = \tilde{\mathbf{B}} + \mathbf{B}_h$  to solve the tearing layer problem in RFP
- uses  $f(\mathbf{v}) \propto \delta(v_{\perp} - v_0) \exp(-v_{\parallel}^2/v_T^2)$ ,  $v_T = 0$
- shows increasing stabilization with increasing  $\rho/a$

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<sup>a</sup>V. A. Svidzinski and S. C. Prager, "Effects of particles with large gyroradii on resistive magnetohydrodynamic stability", PoP **11** 980, 2004

## Simulation Details

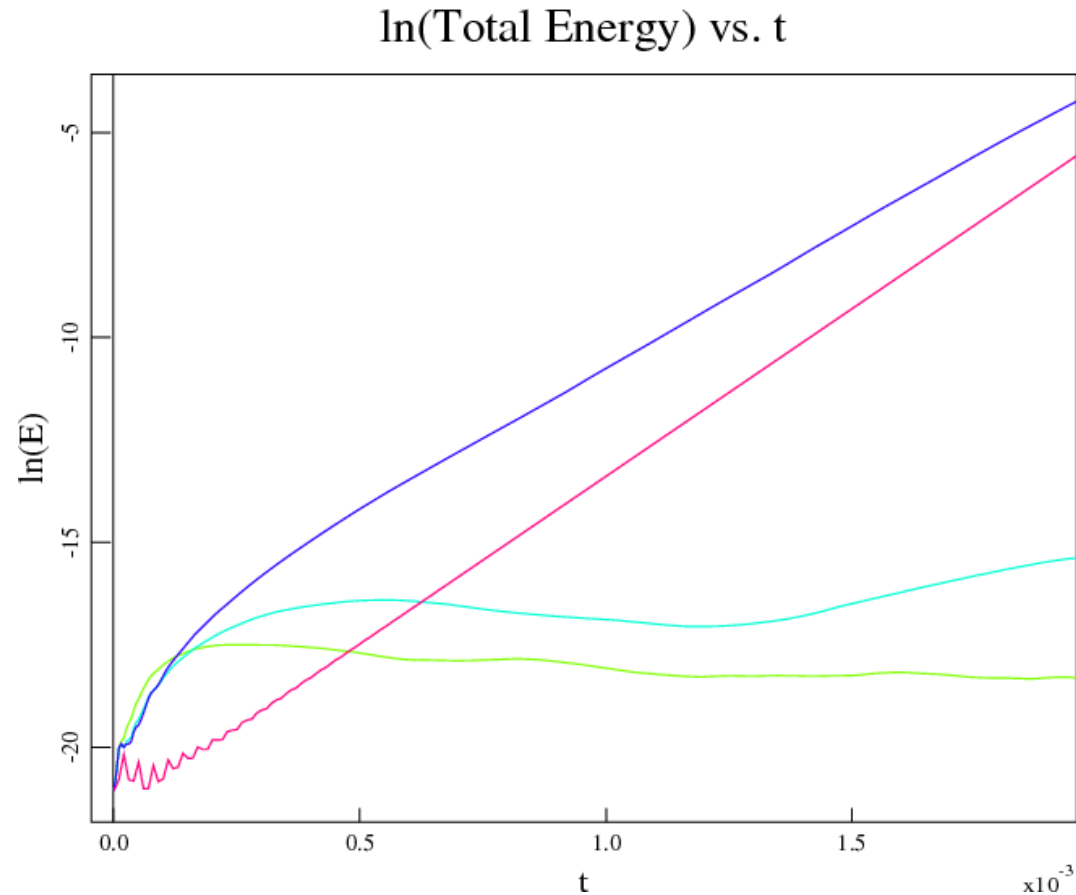
- using  $\delta f$  PIC<sup>a</sup>
- Boris push with orbit averaging
- use only  $\frac{\delta \mathbf{E} + \mathbf{v} \times \delta \mathbf{B}}{B} \cdot \hat{\mathbf{b}} \times \nabla f$  in weight equation
- equilibrium current profile  $\mu = 2\Theta \left[ 1 - \left( \frac{r}{a} \right)^{\alpha_0} \right]$
- energetic ion density profile  $\propto \exp \left[ - \left( \frac{r}{0.45a} \right)^2 \right]$
- in simulation load initial  $\delta(v_{\perp} - v_0)$
- dynamics results in finite spread in  $(v_{\perp}, v_{\parallel})$
- subtract off  $v_{\parallel}$  in calculating weight for  $\mathbf{v} \times \delta \mathbf{B}$  comparison

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<sup>a</sup>S. E. Parker and W. W. Lee, “A fully nonlinear characteristic method for gyrokinetic simulation”, PFB **5**, 77 (1993)

## FLR Stabilization of RFP Tearing Mode

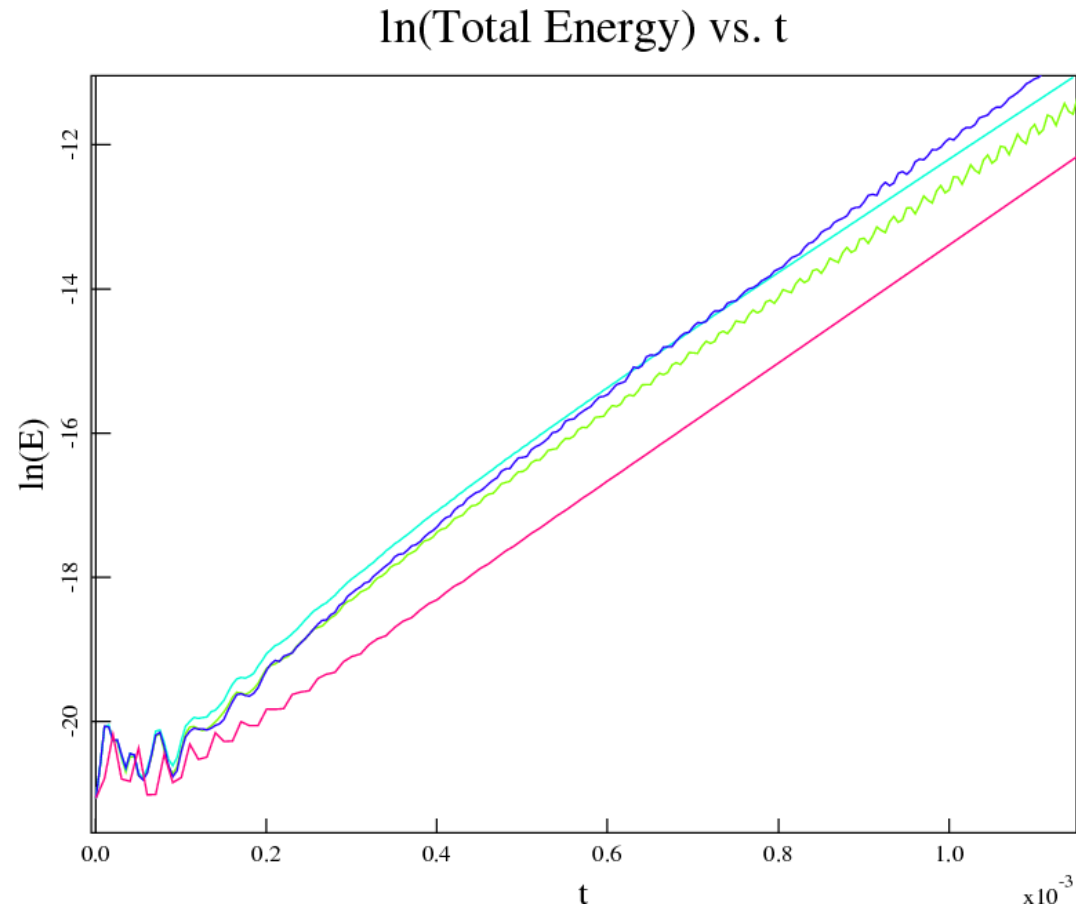
- current profile  $\Theta = 1.75$ ,  $\alpha_0 = 3$ ,  $S = 10^4$  has  $\gamma\tau_A = 1.1e - 3$
- examine effect of increasing Larmor radius  $2\rho_i/a = [.15, .21, .26]$  equivalent to  $v = [1.e6, 1.4e6, 1.8e6]$



- $\mathbf{v} \times \delta\mathbf{B}$  shows strong stabilizing in agreement with theory



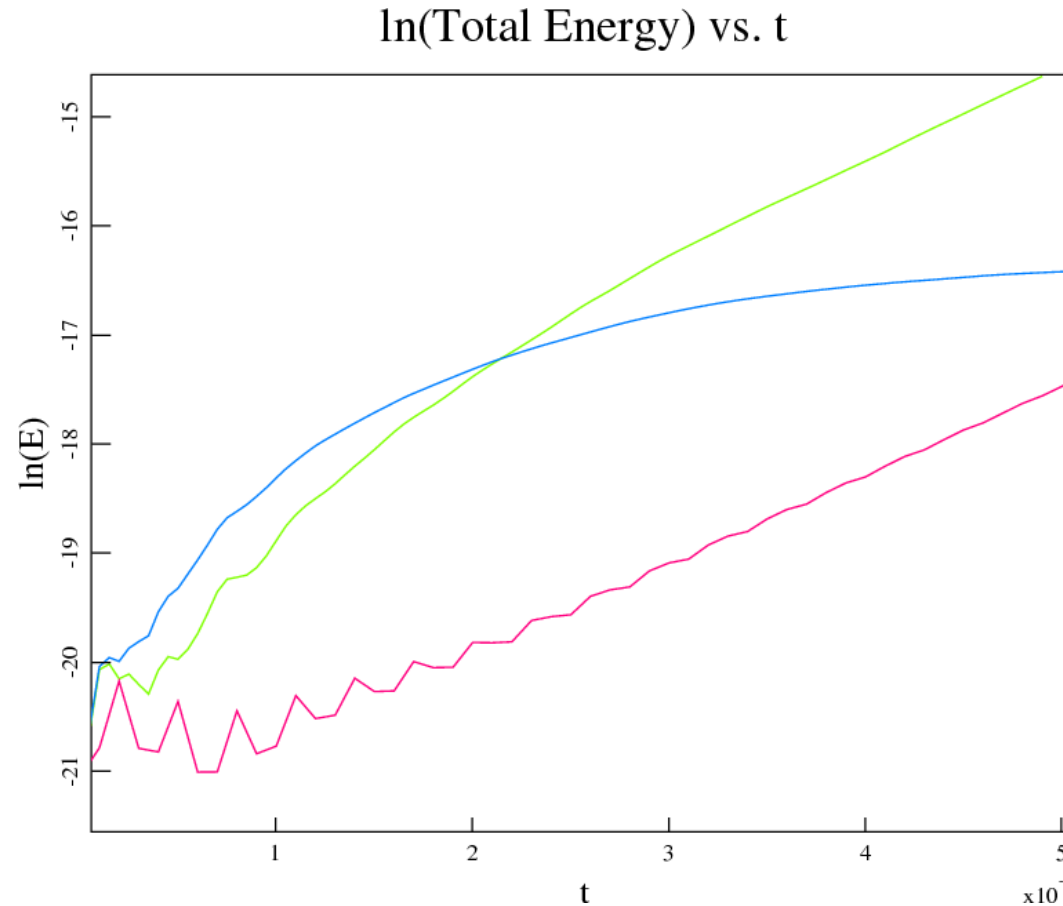
- $\delta E$  negligible effect



- V.Svidzinski points out that finite  $v_{\parallel}$  distribution may reduce effect through phase mixing

## Effects of Velocity Distribution

- Maxwellian distribution with  $v_T = 1.4e6$
- preliminary result from both terms in weight equation



- negligible stabilizing effect

## Summary

- full kinetic particles are implemented and running
- compares well with drift kinetic kink
- shows qualitative agreement with FLR stabilization of tearing mode
- initial runs show that distribution in velocity decreases (possibly eliminates) effect
- preliminary simulations - more analysis needed before conclusions can be drawn
- progress continues - stay tuned